

Homework 5

MATH 5293

1. Can one find a function $F \in H(\mathbb{D})$, continuous on $\overline{\mathbb{D}}$, and such that $F(e^{it}) = \cos t - i \sin t$? Is this possible if \mathbb{D} is replaced above with $\mathbb{C} \setminus \{0\}$?

2. # 10.17, p. 224.

3. Suppose that f is holomorphic in \mathbb{D} , continuous on $\overline{\mathbb{D}}$, and real-valued on an arc $J \subset \partial\mathbb{D}$. Show that f can be extended across J to a holomorphic function satisfying

$$f(z) = \overline{f(1/\bar{z})} \quad \text{for all } z \in G,$$

where G is an open set such that $G \cap \partial\mathbb{D} = J$.

Hint: Use Reflection Principle after a conformal mapping.