Homework 4
MATH 5293

1. # 10.8, p. 188.

2. Let $u : \mathbb{C} \to \mathbb{R}$ be harmonic, and assume that $M \in \mathbb{R}$ is a constant. Prove that $u(z) \leq M$ for all $z \in \mathbb{C}$ implies u is constant. Is the result true if u is bounded below in $\mathbb{C}$? You are not allowed to use holomorphic functions in your solution.

3. Suppose that f is a holomorphic function that satisfies $0 < |f(z)| < 1$ in $D(0, R)$. Show that

$$|f(z)| \leq |f(0)|^{\frac{R-|z|}{R+|z|}} \quad \text{for all } z \in D(0, R).$$

Hint: Consider $u = -\log|f|$.