Homework 2
MATH 5293

1. # 9.6, p. 158.

2. Let $\phi : G \to \mathbb{D}$ be a conformal mapping of a simply connected domain $G \subset \mathbb{C}$ onto the unit disk. Prove that

$$\lim_{z \to t} |\phi(z)| = 1$$

for all $t \in \partial G$.

3. Let $\phi : G \to \mathbb{D}$ be a conformal mapping of a simply connected domain $G \subset \mathbb{C}$ onto the unit disk. Define the sets $K_r = \{z \in G : |\phi(z)| \le r\}$, $r \in [0, 1)$. Show that each K_r is compact, and $K_r \subset (K_R)^\circ$ for all $0 \le r < R < 1$. Construct an exhaustion of G by the sets K_r.

4. Suppose that $G \subset \mathbb{C}$ is a bounded open set, and $f : \mathbb{D} \to G$ is a holomorphic function. Show that the length of the curve $f([0, z])$ is finite for each $z \in \mathbb{D}$. Use a conformal mapping of $\mathbb{D}$ onto an appropriate domain G to give an example of analytic $f : \mathbb{D} \to G$ such that the supremum for the length of $f([0, z])$ over all $z \in \mathbb{D}$ is infinity.

5. Suppose that $G \subset \mathbb{C}$ is a simply connected open set symmetric about the real line, and $a \in G \cap \mathbb{R}$. Let $\phi : G \to \mathbb{D}$ be a conformal mapping such that $\phi(a) = 0$ and $\phi'(a) > 0$. Prove that $\overline{\phi(z)} = \phi(\bar{z})$ for all $z \in G$.