Homework 10
MATH 5293

1. Show that every polynomial has genus and order equal to 0.

2. Let p_n be a polynomial of exact degree n. Show that $f(z) = e^{p_n(z)}$ has genus and order equal to n.

3. (Extra credit) Assume that $\{a_n\}_{n=1}^{\infty} \subset \mathbb{C}$ and $\{A_n\}_{n=1}^{\infty} \subset \mathbb{C}$, where $\lim_{n\to\infty} a_n = \infty$. Prove that there is an entire function f such that $f(a_n) = A_n$ for all $n \in \mathbb{N}$.
Hint: Let g be entire with simple zeros at $\{a_n\}_{n=1}^{\infty}$. Prove that the series

$$\sum_{n=1}^{\infty} \frac{A_n}{g'(a_n)} \frac{e^{c_n(z-a_n)}}{z-a_n} g(z)$$

converges uniformly on compact subsets of $\mathbb{C}$, provided c_n are selected appropriately.

4. (Extra credit) Suppose that the sequence of polynomials $p_n,\ n \in \mathbb{N}$, with non-negative coefficients, converges at every point $z_m = 1/m,\ m \in \mathbb{N}$. Prove that this sequence converges uniformly on compact subsets of $\mathbb{D}$. Can one conclude that p_n converge uniformly on $\overline{\mathbb{D}}$?