1. (3 points) Assume that

\[ \vec{O}A \times \vec{OB} + \vec{OB} \times \vec{OC} + \vec{OC} \times \vec{OA} = \vec{0}. \]

(a) Prove that \( \vec{OA}, \vec{OB} \) and \( \vec{OC} \) are in the same plane.

(b) Prove that \( A, B \) and \( C \) lie on a line.

2. (2 points) Determine whether the following function is continuous at \((0, 0)\):

\[ f(x, y) = \begin{cases} \frac{x^2 y}{x^2 + y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases} \]

3. (4 points) Show that the function

\[ f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0), \end{cases} \]

is continuous and has bounded partial derivatives in a neighborhood of \((0, 0)\), but is not differentiable at \((0, 0)\).

4. (6 points) Prove that the function

\[ f(x, y) = \begin{cases} (x^2 + y^2) \sin \frac{1}{x^2 + y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0), \end{cases} \]

has partial derivatives \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \) that are not continuous at \((0, 0)\).

However, \( f(x, y) \) is differentiable at \((0, 0)\).