MATH 4513 : HOMEWORK 4

1. If

$$\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 4 & 1 & 0 \\ 4 & 5 & 6 & 1 \end{pmatrix} \quad , \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

Write a program that finds the solution of

$$\mathbf{Lx} = \mathbf{b}.$$

•

```
n := 4;
L := array(1..n,1..n);
b := array(1..n);
x := array(1..n);
L := [[1,0,0,0],[2,1,0,0],[3,4,1,0],[4,5,6,1]];
b := [1,2,3,4];
for k from 1 to n do
    x[k] := (b[k] - add(L[k,s]*x[s],s=1..k-1))/L[k,k];
    od;
print(x);
```

```
OUTPUT:  x = [1,0,0,0]
```

□

2. If

$$\mathbf{U} = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad , \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

Write a program that finds the solution of

$$\mathbf{Ux} = \mathbf{b}.$$

•

```
n := 4;
U := array(1..n,1..n);
b := array(1..n);
x := array(1..n);
U := [[1,1,2,1],[0,2,1,2],[0,0,2,1],[0,0,0,1]];
b := [1,2,3,4];
for k from 0 to (n-1) do
    x[n-k] := (b[n-k] - add(U[n-k,n-s]*x[n-s],s=0..k-1))/U[n-k,n-k];
```

```
    od;
  print(x);
```

```
OUTPUT: x = [3/4, -11/4, -1/2, 4]
```

□

3. Write a program to find the LU factorization of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 4 & 4 \\ 1 & 5 & 8 & 8 \\ 2 & 4 & 10 & 14 \end{pmatrix}$$

assuming the lower triangular matrix **L** has 1's along its diagonal.

•

```
n := 4;  # all matrices are nxn=4x4
A := array(1..n,1..n);
L := array(1..n,1..n);
U := array(1..n,1..n);
A := [[1,1,1,1],[2,4,4,4],[1,5,8,8],[2,4,10,14]];

for k from 1 to n do   # calculate kth column of L and kth row of U
    for s from 1 to k-1 do
        L[s,k] := 0;  # so that L is lower triangular
        U[k,s] := 0;  # so that U is upper triangular
        od;
    L[k,k] := 1;       # by convention
    k1 := k-1;
    # calculate the kth element of kth row of U
    U[k,k] := A[k,k] -  sum(L[k,j0]*U[j0,k],j0=1..k1);
    for t from k+1 to n do
        # calculate remaining elements in kth column of L
        L[t,k] := (A[t,k] - add(L[t,j1]*U[j1,k],j1=1..k1))/U[k,k];
        # calculate remaining elements in kth row of U
        U[k,t] := A[k,t] -  add(L[k,j2]*U[j2,t],j2=1..k1);
        od;
     od;
print(L);
print(U);
```

```
OUTPUT :
```

$$\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 2 & 1 & 2 & 1 \end{pmatrix} \quad , \quad \mathbf{U} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

□

4. Write a program to find the LU factorization of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 4 & 4 \\ 1 & 5 & 8 & 8 \\ 2 & 4 & 10 & 14 \end{pmatrix}$$

assuming the upper triangular matrix $\mathbf{U}$ has 1's along its diagonal.

•

```
n := 4;  # all matrices are nxn=4x4
A := array(1..n,1..n);
L := array(1..n,1..n);
U := array(1..n,1..n);
A := [[1,1,1,1],[2,4,4,4],[1,5,8,8], [2,4,10,14]];

for k from 1 to n do   # calculate kth column of L and kth row of U
    for s from 1 to k-1 do
        L[s,k] := 0;  # so that L is lower triangular
        U[k,s] := 0;  # so that U is upper triangular
        od;
    U[k,k] := 1;       # by convention
    k1 := k-1;
    # calculate the kth element of kth row of L
    L[k,k] := A[k,k] -  sum(L[k,j0]*U[j0,k],j0=1..k1);
    for t from k+1 to n do
        # calculate remaining elements in kth column of L
        L[t,k] := (A[t,k] - add(L[t,j1]*U[j1,k],j1=1..k1));
        # calculate remaining elements in kth row of U
        U[k,t] := A[k,t] -  add(L[k,j2]*U[j2,t],j2=1..k1)/L[k,k];
        od;
     od;
print(L);
print(U);
```

```
OUTPUT:
```

$$\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 1 & 4 & -5 & 0 \\ 2 & 2 & 2 & -76/5 \end{pmatrix} \quad , \quad \mathbf{U} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & & 53/5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

□

5. Find the $1 \times 4$ matrix $\mathbf{x}$ that solves

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 4 & 4 \\ 1 & 5 & 8 & 8 \\ 2 & 4 & 10 & 14 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

•

```
n := 4;  # all matrices are nxn=4x4
A := array(1..n,1..n);
L := array(1..n,1..n);
```

```
U := array(1..n,1..n);
A := [[1,1,1,1],[2,4,4,4],[1,5,8,8],[2,4,10,14]];
for k from 1 to n do   # calculate kth column of L and kth row of U
    for s from 1 to k-1 do
        L[s,k] := 0;  # so that L is lower triangular
        U[k,s] := 0;  # so that U is upper triangular
        od:
    L[k,k] := 1;  # by convention
    k1 := k-1;
    # calculate the kth element of kth row of U
    U[k,k] := A[k,k] -  add(L[k,j0]*U[j0,k],j0=1..k1);
    for t from k+1 to n do
        # calculate remaining elements in kth column of L
        L[t,k] := (A[t,k] - add(L[t,j]*U[j,k],j=1..k1))/U[k,k];
        # calculate remaining elements in kth row of U
        U[k,t] := A[k,t] -  add(L[k,j]*U[j,t],j=1..k1);
        od:
    od:
print(`L = `, L);
print(`U = `, U);

# now we solve Lz = b for z
b := array(1..n);
z := array(1..n);
b := [1,2,3,4];
for k from 1 to n do
    k1 := k-1;
    z[k] := (b[k] - add(L[k,s]*z[s],s=1..k1))/L[k,k];
    od:
print(`z = `,  z);

# now we solve Ux = z for x
    x := array(1..n);
    for k from 0 to (n-1) do
        k1 := k-1;
        x[n-k] := (z[n-k] - add(U[n-k,n-s]*x[n-s],s=0..k1))/U[n-k,n-k];
        od:
 print(`x = `, x);
```

OUTPUT:

$$\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 2 & 1 & 2 & 1 \end{pmatrix} \quad , \quad \mathbf{U} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

$$\mathbf{z} = [1, 0, 2, -2]$$

$$\mathbf{x} = [1, -2/3, 7/6, -1/2]$$

4



□