MATH 4513 : HOMEWORK 9

1. Use the Taylor series method to calculate a solution to

$$\frac{dx}{dt} = x^2 t \quad , \quad x(0) = 1$$

that's accurate to order t^4.

2. Use a five step Euler method to calculate the solution of

$$\frac{dx}{dt} = x^2 t \quad , \quad x(0) = 1$$

on the interval $[0, 1]$.

3. Use a five step second order Runge-Kutta method to calculate the solution of

$$\frac{dx}{dt} = x^2 t \quad , \quad x(0) = 1$$

on the interval $[0, 1]$.

4. Use a five step fourth order Runge-Kutta method to calculate the solution of

$$\frac{dx}{dt} = x^2 t \quad , \quad x(0) = 1$$

on the interval $[0, 1]$.

5. Write a Maple program to solve

$$\begin{aligned} \frac{dx}{dt} &= e^{xt} - \cos(x - t) \\ x(1) &= 3 \end{aligned}$$

Use fourth order Runge-Kutta formulas with $h = 0.001$. Stop the computation just before the solution overflows and graph the solution.

6. Derive the second order Runge-Kutta formula

$$x(t + h) = x(t) + hF\left(t + \frac{1}{2}h, x + \frac{1}{2}hF(t, x)\right)$$

by performing a Richardson Extrapolation on Euler's method using step sizes h and $h/2$. (*Hint*: assume the error term is Ch^2.)

7. Derive the third order Runge-Kutta formulas

$$x(t + h) = x(t) + \frac{1}{9}(2F_1 + 3F_2 + 4F_3)$$

where

$$
\begin{aligned}
F_1 &= hF(t, x) \\
F_2 &= hF\left(t + \frac{1}{2}h, x + \frac{1}{2}F_1\right) \\
F_3 &= hF\left(t + \frac{3}{4}h, x + \frac{3}{4}F_2\right)
\end{aligned}
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