MATH 4513 : HOMEWORK 8

1. Write down the Richardson extrapolation for the derivative $f'(x)$ that is accurate to order 8 in h.

2. Suppose

$$\int_a^b f(x)dx$$

is calculated numerically by interpolating the function $f(x)$ at the points

$$x_i = \frac{a+b}{2} + \frac{b-a}{2}\cos\left(\frac{(i+1)\pi}{n+2}\right) \quad , \quad i = 0, 1, 2, \ldots, n$$

and then integrating the interpolation polynomial between a and b. Express the maximal error in terms of a derivative of f, n, and the end points of integration a and b. (Hint: Write down a change of variables formula reduces the integral over $[a, b]$ to an integral over $[-1, 1]$.)

3. Find a quadrature formula for the integral

$$\int_a^{a+3h} f(x)$$

corresponding to the case where the function $f(x)$ is interpolated at four points: $x_0 = a$, $x_1 = a + h$, $x_2 = a + 2h$, $x_3 = a + 3h$;

4. (a) Write a Maple program that applies the formula

$$\Delta x = \frac{b-a}{N}\tilde{x}_i = a + \Delta x i + \frac{\Delta x}{2} \quad , \quad \text{the midpoint of the interval } [x_i, x_{i+1}]$$

to calculate

$$\int_0^2 e^{x^2} dx$$

(take N, the number of subdivsions of $[0, 2]$ equal to 20).

(b) Use the result of Problem 3 to formulate a numerical recipe for calculating

$$\int_a^b f(x)\, dx$$

and then write a Maple program that computes

$$\int_0^2 e^{x^2} dx$$

(take N, the number of subdivsions of $[0, 2]$ equal to 20).