1. If

$$\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 4 & 1 & 0 \\ 4 & 5 & 6 & 1 \end{pmatrix} \quad , \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

Write a program that finds the solution of

 $\mathbf{L}\mathbf{x} = \mathbf{b}$.

2. If

$$\mathbf{U} = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad , \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

Write a program that finds the solution of

 $\mathbf{U}\mathbf{x} = \mathbf{b}$.

3. Write a program to find the LU factorization of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 4 & 4 \\ 1 & 5 & 8 & 8 \\ 2 & 4 & 10 & 14 \end{pmatrix}$$

assuming the lower triangular matrix ${\bf L}$ has 1's along its diagonal.

4. Write a program to find the LU factorization of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 4 & 4 \\ 1 & 5 & 8 & 8 \\ 2 & 4 & 10 & 14 \end{pmatrix}$$

assuming the upper triangular matrix U has 1's along its diagonal.

5. Find the 1×4 matrix **x** that solves

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 4 & 4 \\ 1 & 5 & 8 & 8 \\ 2 & 4 & 10 & 14 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$