MATH 4513 : HOMEWORK 2

1. Let $[a_n, b_n]$ denote the sucessive intervals that arise from applying the bisection method to find a root of a continuous function $f(x)$. Let $c_n = \frac{1}{2}(a_n + b_n)$, $r = \lim_{n\to\infty} c_n$, and $e_n = r - c_n$.

(a) Show that $|e_n| \leq 2^{-n}\,|b-a|$.

(b) Show that $e_n = \mathcal{O}\left(2^{-n}\right)$ as $n \to \infty$.

2. Using the bisection algorithm, find a root of $f(x) = x - \tan(x)$ in the interval $[1, 2]$.

3. Using the bisection algorithm, find a root of $f(x) = 2^{-x} + e^x + 2\cos(x) - 6$ on $[1, 3]$.

4. With a hand held calculator, perform four iterations of Newton's Method to find an approximate zero of

$$f(x) = 4x^3 - 2x^2 + 3$$

starting with $x_0 = -1$.

5. Write a computer program to solve

$$x = \tan x$$

by means of Newton's method. Find the roots nearest 4.5 and 7.7.

6. Write a computer program to solve

$$x^3 + 3x = 5x^2 + 7$$

by Newton's Method. Take ten steps starting with $x_0 = 5$.