MATH 4513 : HOMEWORK 1

1.1. Given that

$$\frac{d^n}{dx^n}\left(\ln|x|\right)\Big|_{x=1} = (-1)^{n-1}(n-1)!$$

(a) Use the Taylor Theorem with Integral Remainder to find the magnitude of the error term $R_{100}(1.99)$ when one approximates $\ln[1.99]$ using the first 101 terms of the Taylor expansion about 1 of $\ln|x|$.

(b) Use the Taylor Theorem with Lagrange Remainder to obtain an upper bound on the error term $E_{100}(x)$ when x ranges from 1.985 to 1.995 for the Taylor expansion of $\ln|x|$ about 1.

1.2. Show that if $b_n = \mathcal{O}(a_n)$ then $b_n/\ln|n| = \mathfrak{o}(a_n)$.

1.3. Show that if $b_n = \mathfrak{o}(a_n)$ then $b_n = \mathcal{O}(a_n)$, but that the converse is not true.

1.4. Show that if $b_n = \mathcal{O}(a_n)$ and $c_n = \mathcal{O}(a_n)$, then $b_n + c_n = \mathcal{O}(a_n)$.

1.5. Show that if $b_n = \mathfrak{o}(a_n)$ and $c_n = \mathfrak{o}(a_n)$, then $b_n + c_n = \mathfrak{o}(a_n)$.

1.6. Show that for any $r > 0$, $x^r = \mathcal{O}(e^x)$ as $x \to \infty$.

1.7. Show that for any $r > 0, \ln|x| = \mathcal{O}(x^r)$ as $x \to \infty$.

1.8. If $1/10$ is correctly rounded to the normalized binary number $(1.a_1a_2\ldots a_{23})_2 \times 2^m$, what is the roundoff error? What is the relative roundoff error?

1.9. Give examples of real numbers for which

$$fl\,(x \odot y) \neq fl\,(fl(x) \odot fl(y))$$

Illustrate this for all four arithmetic operators $(+, -, \times, \div)$ using a hypothetical machine with 5 decimal (base t0) digits.