1. Show that the following ODEs are of the Sturm-Liouville type
\[ \frac{d}{dx} \left[ p(x) \frac{dy}{dx} \right] - q(x) y + \lambda r(x) y = 0, \quad p(x) > 0, \ r(x) > 0 \]
and if so identify the functions \( p(x), q(x) \) and \( r(x) \).

(a) \( y'' + k^2 y = 0 \)

(b) \( x^2 y'' + xy' + (x^2 - \nu^2) y = 0 \)

(c) \( (1 - x^2) y'' = 2xy + \nu (\nu + 1) y = 0 \)

2. (a) Find the Sturm-Liouville eigenfunctions \( \{ \phi_n \} \) for the following Sturm-Liouville system
\[ y'' + \lambda^2 y = 0, \quad y'(0) = 0, \quad y(1) + y'(1) = 0 \]

(b) Suppose \( f(x) \) is a continuous function of \([0, 1]\). Give an integral formula for the coefficients \( a_n \) corresponding to the expansion
\[ f(x) = \sum_{n=1}^{\infty} a_n \phi_n(x) \]
where the functions \( \phi_n(x) \) are the Sturm-Liouville eigenfunctions found in part (a).

3. Solve the following PDE/BVP.
\[ \phi_t - k^2 \phi_{xx} = 0 \]
\[ \phi(x,0) = f(x), \quad 0 \leq x \leq 1 \]
\[ \phi(0,t) = 0, \quad t > 0 \]
\[ \phi(1,t) + \phi_x(1,t) = 0, \quad t > 0 \]