Math 4233
Homework Set 4

1. For each of the following PDEs, try using the method of separation of variables to replace the PDE by a pair of ODEs.

(a) $xu_{xx} + u_t = 0$

(b) $u_{xx} + u_{xt} + u_t = 0$

(c) $tu_{xx} + xu_t = 0$

(d) $[p(x)\, u_x]_x - r(x)\, u_{tt} = 0$

(e) $u_{xx} + u_{yy} + xu = 0$

2. Find the solution of the following heat conduction problem

$$\begin{aligned} 4u_t - u_{xx} &= 0 \quad , \quad 0 < x < 2 \quad , \quad t > 0 \\ u(0,t) &= 0 \\ u(2,t) &= 0 \\ u(x,0) &= 2\sin\left(\frac{\pi x}{2}\right) - \sin(\pi x) + 4\sin(2\pi x) \end{aligned}$$

3. Find the solution of

$$\begin{aligned} 4u_t - u_{xx} &= 0 \quad , \quad 0 < x < 2 \quad , \quad t > 0 \\ u(0,t) &= 2 \\ u(2,t) &= -2 \\ u(x,0) &= 2\sin(\pi x) \end{aligned}$$

4. Show that the wave equation

(*) $$u_{tt} - a^2 u_{xx} = 0$$

can be reduced to the form

$$u_{\xi\eta} = 0$$

by a change for variables $\xi = x - at$, $\eta = x + at$. Conclude that the any solution of (*) can be written as

$$u(x,t) = \phi(x - at) + \psi(x + at) \quad .$$

5. Find the solution of Laplace's equation

$$u_{xx} + u_{yy} = 0$$

satisfying the boundary conditions

$$\begin{aligned} u(x,0) &= 0 \quad , \quad u(x,b) = g(x) \\ u(0,y) &= 0 \quad , \quad u(a,y) = 0 \end{aligned}$$

6. Express the 2-dimensional Laplace equation

$$u_{xx} + u_{yy} = 0$$

in terms of polar coordinates (r, θ) and use separation of variables to reduce it to the solution of a pair of ordinary differential equations.