Math 4233
Homework Set 1

1. Find the inverses of the following matrices:

(a) $\begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

2. Find the eigenvectors and eigenvalues of the following matrices:

(a) $\begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix}$

3. For each of the matrices $\mathbf{A}$ in Problem 2 find a diagonal matrix $\mathbf{D}$ and an invertible matrix $\mathbf{C}$ such that
$$\mathbf{D} = \mathbf{C}^{-1}\mathbf{A}\mathbf{C}$$

4. For each of the following systems find the fundamental (independent) solutions.

(a) $\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} \mathbf{x}$

(b) $\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix} \mathbf{x}$

(c) $\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -8 & -5 & -3 \end{bmatrix} \mathbf{x}$

5. Find the general solution of the following systems:

(a) $\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \mathbf{x}$

(b) $\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \mathbf{x}$