MATH 4063-5023
Homework Set 7

1. Find the characteristic polynomials and minimal polynomials of the following matrices.

(a) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

2. Find the eigenvalues of the following matrice, and then for each eigenvalue, find a basis for the corresponding eigenspace, and state the algebraic and geometric multiplicity of eigenvalue.

(a) $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 2 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} -3 & 0 & 4 \\ 0 & -1 & 0 \\ -2 & 7 & 3 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ (you'll have to work over $\mathbb{C}$ for this one.)

3. Let V be a vector space with basis $\{v_1, \ldots, v_m\}$ .Find a basis for $L(V, V)$.

4. Suppose V is an n-dimensional vector space and $T : V \to V$ is an endomorphims of V with n linearly independent eigenvectors $v_1, \ldots, v_n$ with eigenvalues $\xi_1, \ldots, \xi_n$. Set

$$f(x) = \prod_{i=1}^{n} (x - \xi_i)$$

(a) Show $f(T) = 0$.

(b) Show the minimal polynomial of T is the product of the distinct factors $(x - \xi_i)$ of f.

5. Show that $T \in L(V, V)$ is invertible if and only if the constant term of the minimal polynomial is not equal to zero. Come up with an algorithm for computing T^{-1} from its minimal polynomial.

6. Two matrices $\mathbf{A}$ and $\mathbf{B}$ are said to be *similar* if there exists an invertible matrix $\mathbf{C}$ such that $\mathbf{B} = \mathbf{C}^{-1}\mathbf{A}\mathbf{C}$. Show that similar matrices have the same eigenvalues.

7. Suppose $T \in L(V, V)$ has $n = \dim V$ distinct eigenvalues. Show that there exists a basis of V consisting of eigenvectors of T. What will be the matrix of T with respect to this basis?