MATH 4063-5023
Homework Set 6

1. Let $B_1 = \{[1,1],[1,-1]\}$ and let $B_2 = \{[3,7],[-1,-3]\}$. Regarding B_1 and B_2 as bases for $\mathbb{R}^2$, find the change-of-coordinates-matrix that converts coordinate vectors with respect to B_1 to coordinate vectors w.r.t. B_2.

2. Let $B_1 = \left\{1, x, x^2\right\}$ and let $B_2 = \left\{1, x-1, (x-1)^2\right\}$. Regarding B_1 and B_2 as bases for the vector space of polynomials of degree ≤ 2, find the change-of-coordinates-matrix that converts coordinate vectors with respect to B_1 to coordinate vectors with respect to B_2.

3. Use the definition $\det(\mathbf{M}) = \sum_{\sigma \in S_n} \varepsilon(\sigma) M_{1\sigma_1} \cdots M_{2\sigma_2}$ to calculate the determinant of $\mathbf{M} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$

4. Consider the following matrix

$$\mathbf{M} = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 2 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 1 \end{pmatrix}$$

(a) Use row reduction to calculate the determinant of $\mathbf{M}$.

(b) Use a cofactor expansion to calculate the determinant of $\mathbf{M}$.

5. Determine if the vectors $\mathbf{v}_1 = [0,1,2,1]$, $\mathbf{v}_2 = [1,0,0,2]$, $\mathbf{v}_3 = [2,1,1,1]$ and $\mathbf{v}_4 = [0,0,1,0]$ are linearly independent by calculating a particular determinant.

6. Consider the matrix

$$\mathbf{M} = \begin{pmatrix} 3 & 0 & 4 \\ -2 & 1 & 1 \\ 3 & 1 & 2 \end{pmatrix}$$

(a) Compute the cofactor matrix $\mathbf{C_M}$ of $\mathbf{M}$.

(b) Use the result of 7(a) to compute $\mathbf{A}^{-1}$.

7. Solve the following system of linear equations using Crammer's Rule.

$$\begin{aligned} x_1 + 2x_2 - x_3 &= -3 \\ 2x_1 + x_2 + x_3 &= 0 \\ 3x_1 - x_2 + 5x_3 &= 1 \end{aligned}$$