MATH 4063-5023

Homework Set 5

1. Suppose that V is a finitely generated vector space and $\phi : V \to W$ is a linear transformation. Show that $Range\,(\phi) \subset W$ is finitely generated.

2. Let S be a subspace of V and let $p_S : V \to V/S$ be the canonical projection: $p_S\,(v) = [v]_S$. Show that p_S is a linear transformation with kernel S and image V/S.

3. Suppose S is a subspace of a finitely generated vector space V. Show that V/S is finitely generated.

4. Let V be a finitely generated vector space and let S be a subspace of V. Let $B_S = \{b_1, \ldots, b_m\}$ be a basis for S and let $B_V = \{b_1, \ldots, b_m, b_{m+1}, \cdots, b_n\}$ be a basis for V obtained by extending B_S to a basis for V (see Theorem 5.4). Let $p_S : V \to V/S$ be the canonical projection. Show that $\{p_S\,(b_{m+1}), \ldots, p_S\,(b_n)\}$ is a basis for V/S.

5. Suppose S is a subspace of a finitely generated vector space V, show that $\dim (V) = \dim (S) + \dim (V/S)$.

6. Suppose $\phi : V \to W$ is a linear transformation between two finite-dimensional vector spaces. Show that $Range\,(\phi)$ is isomorphic to $V/\ker(\phi)$. (Hint: two finite-dimensional vector spaces are isomorphic if and only if they have the same dimension.)

7. Let S be the subspace of $\mathbb{R}^3$ spanned by $[1, 0, 0]$ and $[0, 1, 0]$. Identify let $\mathbf{v}_1 = [1, -1, 3]$ and let $\mathbf{v}_2 = [2, 3, 1]$. Determine $[\mathbf{v}_1]_S + [\mathbf{v}_2]_S$ explicitly (it has to be some hyperplane in the direction of S inside $\mathbb{R}^3$).