MATH 4063-5023
Homework Set 3

1. Test for the solvability of the following linear systems (over $\mathbb{R}$). If the system is solvable, then express the general solution in the form of $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_0$ where $\mathbf{x}_0$ is a particular solution of the given linear system and $\mathbf{x}_0$ is the general solution of the corresponding homogeneous linear system (see the tail end of Lecture 6).

(a)

$$\begin{aligned} x_1 + x_2 + x_3 &= 8 \\ x_1 + x_2 + x_4 &= 1 \\ x_1 + x_3 + x_4 &= 14 \\ x_2 + x_3 + x_4 &= 14 \end{aligned}$$

(b)

$$\begin{aligned} 2x_1 + x_2 + 3x_3 - x_4 &= 1 \\ 3x_1 + x_2 - 2x_3 + x_4 &= 0 \\ 2x_1 + x_2 - x_3 + 2x_4 &= -1 \end{aligned}$$

(c)

$$\begin{aligned} x_1 + 4x_2 + 3x_3 &= 1 \\ 3x_1 + x_3 &= 1 \\ 4x_1 + x_2 + 2x_3 &= 1 \end{aligned}$$

(d)

$$\begin{aligned} -x_1 + 2x_2 + x_3 + 4x_4 &= 0 \\ 2x_1 + x_2 - x_3 + x_4 &= 1 \end{aligned}$$

2. Prove that an $n \times m$ system of homogeneous equations has a non-trivial solution if and only if the rank of the coefficient matrix is less than m.

3. Find a set of homogeneous linear equations whose solution set is the subspace of $\mathbb{R}^3$ generated by the vectors $[2, 1, -3], [1, -1, 0]$ and $[1, 3, -4]$.

4. Let S be a subspace of a vector space V. Prove that if $\mathbf{p}$ and $\mathbf{q}$ are vectors belonging to the hyperplane $M = H_{\mathbf{x}_0, S} = \{\mathbf{x}_0 + \mathbf{s} \mid \mathbf{s} \in S\}$, then the line through $\mathbf{p}$ and $\mathbf{q}$ lies entirely in M.

5. Find the point in $\mathbb{R}^3$ where the line joining the points $[1, -1, 0]$ and $[-2, 1, 1]$ pierces the plane $3x_1 - x_2 + x_3 = 1$.