MATH 4063-5023
Homework Set 2

1. Use elementary row operations to systematically transform the following matrices to row echelon form.

(a) $\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 3 & 0 & -1 \\ 1 & 2 & 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & -1 \\ 1 & 2 & 4 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & -1 & 1 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 3 & -1 & -4 \end{pmatrix}$

2. Test the following sets of vectors for linear independence.

(a) $\{[-1,1], [1,2], [1,3]\}$

(b) $\{[0,1,1,2], [3,1,5,2], [-2,1,0,1], [1,0,3,-1]\}$

(c) $\{[1,1,0,0,1], [-1,1,1,0,0], [2,1,0,1,1], [0,-1,-1,-1,0]\}$

3. Test the following sets of polynomials for linear dependence.

(a) $\left\{x^2+2x+1, 2x+1, 2x^2-2x-1\right\}$

(b) $\left\{1, x-1, (x-1)^2, (x-1)^3\right\}$

4. Determine if $[1,1,1]$ belongs to the subspace of $\mathbb{R}^3$ generated by $[1,3,4], [4,0,1], [3,1,2]$. Explain your reasoning.

5. Prove every subspace S of a finitely generated vector space T is finitely generated and that $\dim S \leq \dim T$ with equality only if $S = T$.

6. Let $\mathbb{F}$ be a field with exactly two elements (it will be isomorphic to $\mathbb{Z}_2$) and let V be a 2-dimensional vector space over $\mathbb{F}$. How many vectors are there in V? How many different bases are there for V?