Math 4023
Homework Set 8

1. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2 - 3x + 5$. Use the definition of continuity to prove that f is continuous at $x = 2$.

2. Let $f : D \to \mathbb{R}$ be continuous. For each of the following either prove or give a counter-example.

   (a) If D is open, then $f(D)$ is open.
   (b) If D is closed, then $f(D)$ is closed.
   (c) If D is not open, then $f(D)$ is not open.
   (d) If D is not closed, then $f(D)$ is not closed.
   (e) If D is not compact, then $f(D)$ is not compact.
   (f) If D is unbounded, then $f(D)$ is unbounded.
   (g) If D is infinite, then $f(D)$ is infinite.
   (h) If D is an interval, then $f(D)$ is an interval.
   (i) If D is an interval that is not open, then $f(D)$ is an interval that is not open.

3. Show that any polynomial of odd degree has one at least real root.

4. Suppose $f : [a, b] \to [a, b]$ is continuous. Prove that f has a fixed point c; i.e., a point $c \in [a, b]$ such that $f(c) = c$.

5. Suppose $f : D \to E$ and $g : E \to F$ are continuous. Show that $g \circ f : D \to F$ is continuous.

6. Use the definition of the derivative to compute the derivative of $f(x) = 2x + 7$ for $x \in \mathbb{R}$.