Math 4023
Homework Set 2

1. Let f and g be functions.1. Suppose f is a function from A to B and let C_1, C_2 be subsets of A and D_1, D_2 be subsets of D. Prove the following identities:

(a) $f(C_1 \cap C_2) \subset f(C_1) \cap f(C_2)$
(b) $f(C_1 \cup C_2) = f(C_1) \cup f(C_2)$
(c) $f^{-1}(D_1 \cap D_2) = f^{-1}(D_1) \cap f^{-1}(D_2)$

2 . Suppose f is a function from A to B and let C and D be subsets of, respectively, A and B. Prove the following

1. If f is injective, then $f^{-1}(f(C)) = C$.
2. If f is surjective, then $f\left(f^{-1}(D)\right) = D$.

3. Suppose $f : A \to B$ and $g : B \to C$. Prove that if f and g are injective, then $g \circ f$ is injective.

4. In each part, find a function $f : \mathbb{N} \to \mathbb{N}$ with the following properties

(a) f is surjective but not injective.
(b) f is injective but not surjective.
(c) f is neither injective or surjective.
(d) f is bijective.

5. Show that each of the following pairs of sets are equinumerous by finding a specific bijection between them.

(a) $S = [0, 1]$ and $T = [1, 3]$
(b) $S = [0, 1]$ and $T = [0, 1)$
(c) $S = [0, 1)$ and $T = (0, 1)$
(d) $S = (0, 1)$ and $T = (0, \infty)$
(e) $S = (0, 1)$ and $T = \mathbb{R}$

6. Prove that if S is denumerable, then S is equinumerous with a proper subset with itself.

7. Prove that every infinite set has a denumerable subset.

8. Prove that every infinite set is equinumerous with a proper subset of itself.