Math 4023
Homework Set 1

1. Prove, by the forward-backward method, that if m and n are even integers, then $n+m$ is even.

2. Prove, using proof-by-contradiction, that if n is an odd integer, then n^2 is an odd integer.

3. Prove, by the contrapositive method, that if n is an integer and n^2 is odd, then n is odd.

4. Prove, by the contrapositive method, that if c is an odd integer then the equation n^2+n-c has no integer solution for n.

5. Prove, by mathematical induction, that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad .$$

6. Prove the following identities:

   (a) $B \cap (C \cup D) = (B \cap C) \cup (B \cap D)$
   (b) $B \cup (C \cap D) = (B \cup C) \cap (B \cup D)$
   (c) $C = (C - A) \cup (C \cap A)$

7. Let $B = \{1,2,3,4\}$ and $C = \{a,b,c\}$

   (a) List four different surjective functions from B to C.
   (b) List four different injective functions from C to B¿
   (c) List all bijective functions from C to C.
   (e) Give an example of a function that is injective but not surjective.
   (f) Give an example of a function that is surjective but not injective.

9. Let B and C be nonempty sets. Prove that the function

$$f : B \times C \to C \times B$$

given by $f(x,y) = (y,x)$ is a bijection.