Math 4013
Homework Problems from Chapter 7

Section 7.1

7.1.1. Evaluate $\int_\sigma f\,ds$ where $f(x,y,z) = x+y+z$ and $\sigma : t \mapsto (\sin(t),\cos(t),t)$, $t \in [0,2\pi]$.

7.1.2. Evaluate the path integral $\int_\mathcal{C} f\,ds$ where $f(x,y,z) = yz$ and $\mathcal{C}$ is the curve parameterized by $\sigma : t \mapsto (t,3t,2t)$, $t \in [1,3]$.

Section 7.2

7.2.1. Let $\mathbf{F}(x,y,z) = x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$. Evaluate the line integral of $\mathbf{F}$ along the path $\sigma(t) = (t,t,t)$, $0 \le t \le 1$.

7.2.2. Consider the force $\mathbf{F}(x,y,z) = x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$. Compute the work done in moving along the parabola $y = x^2$, $z = 0$, from $x = -1$ to $x = 2$.

Section 7.3

7.3.1. Find the equation of the tangent plane to the parameterized surface $\Phi(u,v) = \left(2u, u^2+v, v^2\right)$ at the point (0,1,1).

7.3.2. Find an expression for the unit vector normal to the parameterized surface

$$\Phi(u,v) = (\cos(v)\sin(u), \sin(v)\sin(u), \cos(u)) \quad , \quad (u,v) \in [0,\pi] \times [0,2\pi] \quad .$$

Identify this surface.

Section 7.4

7.4.1. Find the surface area of the unit sphere S represented parametrically by

$$\Phi(\theta,\phi) = (\cos(\theta)\sin(\phi), \sin(\theta)\sin(\phi), \cos(\phi)) \quad , \quad (\theta,\phi) \in [0,2\pi] \times [0,\pi] \quad .$$

7.4.2. Let $\Phi(u,v) = (u-v, u+v, uv)$ and let D be the unit disk in the uv plane. Find the area of $\Phi(D)$.

Section 7.5

7.5.1. Evaluate $\int_S z\,dS$ where S is the upper hemisphere of radius a, that is, the set

$$\left\{(x,y,z) \in \mathbb{R}^3 \mid z = \sqrt{a^2+x^2+y^2}\right\} \quad .$$

Section 7.6

7.6.1. Let the temperature of a point in $\mathbb{R}^3$ be given by $3x^2+3z^2$. Compute the heat flux across the surface $x^2+z^2 = 2$, $0 \le y \le 2$ if $k = 1$