Math 4013
Homework Problems from Chapter 5

Section 5.2

5.2.1. Evaluate the following interated integrals.

(a)

$$\int_{-1}^{1}\int_{0}^{1}\left(x^4y+y^2\right)dy\,dx$$

(b)

$$\int_{0}^{\pi/2}\int_{0}^{1}\left(y\cos(x)+2\right)dy\,dx$$

5.2.2. Evaluate the integrals in 5.1.1 by integrating first with respect to x and then with respect to y.

5.2.3. (a) Demonstrate informally that the volume of the solid of revolution shown in Figure 5.1.13. is

$$\pi\int_{a}^{b}[f(x)]^2\,dx \quad .$$

(b) Show the volume of the region obtained by rotating the region under the graph of parabola $y = -x^2+2x+3$, $-1 \le x \le 3$, about the x-axis is $512\pi/15$.

5.2.4. Evaluate the following double integrals

(a)

$$\int_R \left(x^2y^2+x\right)dxdy \quad , \quad R=[0,2]\times[-1,0]\,.$$

(b)

$$\int_R \left(x^3+y^3\right)dA \quad , \quad R=[0,1]\times[0,1]\,.$$

(c)

$$\int_R ye^{xy}\,dA \quad , \quad R=[0,1]\times[0,1]\,.$$

(d)

$$\int_R \left(x^my^n\right)dA \quad , \quad m,n>0 \quad , \quad R=[0,1]\times[0,1]\,.$$

(e)

$$\int_R \left(ax+by+c\right)dA \quad , \quad R=[0,1]\times[0,1]\,.$$

5.2.5. Compute the volume of the solid bounded by the surface $z=\sin(y)$, the planes $x=1$, $x=0$, $y=0$, $y=\frac{\pi}{2}$, $z=0$.

Section 5.3

5.3.1(a). Evaluate the following iterated integral and draw the region D determined by the limits of integration. State whether the region D is of type I, type II, or both.

$$\int_0^1 \int_0^{x^2} dy dx$$

5.3.1(b). Evaluate the following iterated integral and draw the region D determined by the limits of integration. State whether the region D is of type I, type II, or both.

$$\int_0^1 \int_1^{e^x} (x+y) dy\, dx$$

5.3.2. Use double integrals to compute the area of a circle of radius r.

5.3.3. Let D be the region bounded by the x and y axes and the line $3x + 4y = 10$. Compute

$$\int_D \left(x^2 + y^2\right) dA \quad .$$

5.3.4 . Let $D = \left\{(x, y) \in \mathbb{R}^2 \mid 1 \leq x^2 + y^2 \leq 2 \quad , \quad y \geq 0\right\}$. Is D an elementary region? Evaluate

$$\int_D (1 + xy)\, dA \quad .$$

Section 5.4

5.4.1. Change the order of integration, sketch the corresponding region, and evaluate the following integrals both ways.

(a)

$$\int_0^1 \int_x^1 (xy) dy\, dx$$

(b)

$$\int_0^{\frac{\pi}{2}} \int_0^{\cos(\theta)} \cos(\theta) dr\, d\theta \quad .$$

(c)

$$\int_0^1 \int_1^{2-y} (x+y)^2\, dx dy$$

5.4.2. Compute the volume of the ellipsoid with semiaxes a, b, and c. (Hint: use symmetry and first find the volume of half the ellipsoid.)

5.4.3. Evaluate

$$\int_D e^{x-y} dA$$

where D is the interior of the triangle with vertices (0,0), (1,3) and (2,2).

5.4.4. Evaluate

$$\int_D y^3 \left(x^2 + y^2\right)^{-3/2} dA$$

where D is the region determined by the conditions $\frac{1}{2} \leq y \leq 1$ and $x^2 + y^2 \leq 1$.

Section 5.6

5.6.1. Evaluate

$$\int_W x^2 dV$$

where $W = [0,1] \times [0,1] \times [0,1]$.

5.6.2. Evaluate

$$\int_W ye^{-xy} dV$$

where $W = [0,1] \times [0,1] \times [0,1]$.

5.6.3. Evaluate

$$\int_W (2x + 3y + z) dV$$

where $W = [1,2] \times [-1,1] \times [0,1]$.

5.6.4. Evaluate

$$\int_0^1 \int_0^{2x} \int_{x^2+y^2}^{x+y} dz\, dy\, dx$$

and sketch the region of integration.

5.6.6. Compute the integral of the function $f(x,y,z) = z$ over the region W in the first octant of $\mathbb{R}^3$ bounded by the planes $y = 0$, $z = 0$, $x + y = 2$, $2y + x = 6$, and the cylinder $y^2 + z^2 = 4$.

5.6.7. Evaluate

$$\int_S xyz\, dV$$

where S is the region determined by the conditions $x \geq 0$, $y \geq 0$, $z \geq 0$, and $x^2 + y^2 + z^2 \leq 1$.