LECTURE 24

Group Homomorphisms

DEFINITION 24.1. *A map $f : G \to H$ be a map between two groups is called a* ***group homomorphism*** *if*

$$f(g * g') = f(g) * f(g') \quad , \quad \forall\, g, g' \in G \quad .$$

A group homorphism $f : G \to H$ is said to be an ***group isomorphism*** *if the map f is bijective. Two groups G and H are said to be* ***isomorphic*** *(written $G \cong H$) if there exists an isomorphism $f : G \to H$.*

THEOREM 24.2. *Every infinite cylic group is isomorphic to $\mathbb{Z}$. Every finite cyclic group of order n is isomorphic to $\mathbb{Z}_n$.*

THEOREM 24.3. *Let G and H be groups with identity elements e_G and e_H, respectively. If $f : G \to H$ is a homomorphism, then ı(i) $f(e_G) = e_H$. ı(ii) $f\left(a^{-1}\right) = (f(a))^{-1}$ for every $a \in G$. ı(iii) $Im[f]$ is a subgroup of H. ı(iv) If f is injective, then $G \cong Im[f]$.*

THEOREM 24.4 (Cayley's Theorem). *Every group G is isomorphic to a subgroup of a permutation group.*

COROLLARY 24.5. *Every finite group G of order n is isomorphic to a subgroup of the symmetric group S_n.*