1. Prove that “not- \( Q \Rightarrow \) not-\( P \)” implies “\( P \Rightarrow Q \)”

2. Prove that if \( m \) and \( n \) are even integers, then \( n + m \) is an even integer.

3. Prove that if \( n \) is an odd integer, then \( n^2 \) is an odd integer.

4. Prove that if \( n \) is an integer and \( n^2 \) is odd, then \( n \) is odd.

5. Prove, by the contrapositive method, that if \( c \) is an odd integer then the equation \( n^2 + n - c = 0 \) has no integer solution.

6. Prove, by mathematical induction, that if \( n \geq 5 \) then \( 2^n > n^2 \).

7. Prove by mathematical induction that

\[
\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \forall n \in \mathbb{Z}^+.
\]

8. Prove the following identities.

(a) \( B \cap (C \cup D) = (B \cap C) \cup (B \cap D) \)

(b) \( B \cup (C \cap D) = (B \cup C) \cap (B \cup D) \)

(c) \( C = (C - A) \cup (C \cap A) \)

9. Describe each set in set-builder notation:

(a) All positive real numbers.

(b) All negative irrational numbers.

(c) All points in the coordinate plane with rational first coordinate.

(d) All negative even integers greater than \(-50\).

10. Which of the following sets are nonempty?

(a) \( \{ r \in \mathbb{Q} \mid r^2 = 2 \} \)

(b) \( \{ r \in \mathbb{R} \mid r^2 + 5r - 7 = 0 \} \)

(c) \( \{ t \in \mathbb{Z} \mid 6t^2 - t - 1 = 0 \} \)
11. Is $B$ is a subset of $C$ when

(a) $B = \mathbb{Z}$ and $C = \mathbb{Q}$?
(b) $B$ = all solutions of $x^2 + 2x - 5 = 0$ and $C = \mathbb{Z}$?
(c) $B = \{a, b, 7, 9, 11, -6\}$ and $C = \mathbb{Q}$?

12. In each part find $B - C$, $B \cap C$, and $B \cup C$.

(a) $B = \mathbb{Z}$ and $C = \mathbb{Q}$.
(b) $B = \mathbb{R}$ and $C = \mathbb{Q}$.
(c) $B = \{a, b, c, 1, 2, 3, 4, 5, 6\}$ and $C = \{a, c, e, 2, 4, 6, 8\}$.

13. Let $A, B$ be subsets of $U$. Prove De Morgan’s laws:

(a) $U - (A \cap B) = (U - A) \cup (U - B)$
(b) $U - (A \cup B) = (U - A) \cap (U - B)$

14. (a) Give an example of a function that is injective but not surjective.
(b) Give and example of a function that is surjective but not injective.

15. Prove that $f : \mathbb{R} \to \mathbb{R} : f(x) = 2x + 1$ is injective.

16. Prove that $f : \mathbb{R} \to \mathbb{R} : f(x) = -3x + 5$ is surjective.

17. Let $B$ and $C$ be nonempty sets. Prove that the function

$f : B \times C \to C \times B$

given by $f(x, y) = (y, x)$ is a bijection.