MATH 3613
Homework Set 1

1. Prove that "not-$Q \Rightarrow$ not-P" implies "$P \Rightarrow Q$"

2. Prove that if m and n are even integers, then $n+m$ is an even integer.

3. Prove that if n is an odd integer, then n^2 is an odd integer.

4. Prove that if n is an integer and n^2 is odd, then n is odd.

5. Prove, by the contrapositive method, that if c is an odd integer then the equation $n^2+n-c=0$ has no integer solution.

6. Prove, by mathematical induction, that if $n \geq 5$ then $2^n > n^2$.

7. Prove by mathematical induction that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad , \qquad \forall\, n \in \mathbb{Z}^+ \quad .$$

8. Prove the following identities.

(a) $B \cap (C \cup D) = (B \cap C) \cup (B \cap D)$

(b) $B \cup (C \cap D) = (B \cup C) \cap (B \cup D)$

(c) $C = (C - A) \cup (C \cap A)$

9. Describe each set in set-builder notation:

(a) All positive real numbers.

(b) All negative irrational numbers.

(c) All points in the coordinate plane with rational first coordinate.

(d) All negative even integers greater than -50.

10. Which of the following sets are nonempty?

(a) $\left\{r \in \mathbb{Q} \mid r^2 = 2\right\}$

(b) $\left\{r \in \mathbb{R} \mid r^2 + 5r - 7 = 0\right\}$

(c) $\left\{t \in \mathbb{Z} \mid 6t^2 - t - 1 = 0\right\}$

11. Is B is a subset of C when

(a) $B = \mathbb{Z}$ and $C = \mathbb{Q}$?

(b) $B =$ all solutions of $x^2 + 2x - 5 = 0$ and $C = \mathbb{Z}$?

(c) $B = \{a, b, 7, 9, 11, -6\}$ and $C = \mathbb{Q}$?

12. In each part find $B - C$, $B \cap C$, and $B \cup C$.

(a) $B = \mathbb{Z}$ and $\mathbb{C} = \mathbb{Q}$.

(b) $B = \mathbb{R}$ and $\mathbb{C} = \mathbb{Q}$.

(c) $B = \{a, b, c, 1, 2, 3, 4, 5, 6\}$ and $C = \{a, c, e, 2, 4, 6, 8\}$.

13. Let A, B be subsets of U. Prove De Morgan's laws:

(a) $\qquad U - (A \cap B) = (U - A) \cup (U - B)$

(b) $\qquad U - (A \cup B) = (U - A) \cap (U - B)$

14.
(a) Give an example of a function that is injective but not surjective.

(b) Give and example of a function that is surjective but not injective.

15. Prove that $f : \mathbb{R} \to \mathbb{R} \quad : \quad f(x) = 2x + 1$ is injective.

16. Prove that $f : \mathbb{R} \to \mathbb{R} \quad : \quad f(x) = -3x + 5$ is surjective.

17. Let B and C be nonempty sets. Prove that the function

$$f : B \times C \ \to \ C \times B$$

given by $f(x, y) = (y, x)$ is a bijection.