1. Write down the formal definitions of the following notions:
(a) a linear transformation from \(\mathbb{R}^m\) to \(\mathbb{R}^n\)
(b) the range of a linear transformation \(T : \mathbb{R}^m \rightarrow \mathbb{R}^n\)
(c) the kernel of a linear transformation \(T : \mathbb{R}^m \rightarrow \mathbb{R}^n\)

2. Consider the following mapping: \(T : \mathbb{R}^3 \rightarrow \mathbb{R}^2 : T ([x_1, x_2, x_3]) = [x_2, x_1 - x_3]\). Show that \(T\) is a linear transformation.

3. Suppose \(T\) is the linear transformation from \(\mathbb{R}^3\) to \(\mathbb{R}^4\) given by
\[T ([x_1, x_2, x_3]) = [x_1 + x_2, -x_1 + x_3, x_2 + x_3, 0]\]
(a) Find the matrix \(A_T\) such that \(T (x) = Ax\) for all \(x \in \mathbb{R}^3\).
(b) Find a basis for the range of \(T\)
(c) Find a basis for the kernel of \(T\).

4. Compute the following determinants by the indicated method
(a) \(\det \begin{pmatrix} 3 & 3 & 0 & 2 \\ 0 & 2 & 0 & 1 \\ 1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}\) via row reduction
(b) \(\det \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix}\) via a cofactor expansion

5. For each of the matrices \(A\) below
   • Find the eigenvalues of \(A\)
   • Find the eigenvectors of \(A\)
   • Determine the both algebraic multiplicities and geometric multiplicity of each eigenvalue
(a) \(A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}\)
(b) \(A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\)