Math 3013

SAMPLE SECOND EXAM

1. Write down the formal definitions of the following notions:

(a) a *linear transformation from* $\mathbb{R}^m$ to $\mathbb{R}^n$

(b) the *range* of a linear transfomation $T : \mathbb{R}^m \to \mathbb{R}^n$

(c) the *kernel* of a linear transformation $T : \mathbb{R}^m \to \mathbb{R}^n$

2. Consider the following mapping: $T : \mathbb{R}^3 \to \mathbb{R}^2 : T([x_1, x_2, x_3]) = [x_2, x_1 - x_3]$. Show that T is a linear transformation.

3. Suppose T is the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^4$ given by

$$T([x_1, x_2, x_3]) = [x_1 + x_2 \ , \ -x_1 + x_3 \ , \ x_2 + x_3 \ , \ 0]$$

(a) Find the matrix $\mathbf{A}_T$ such that $T(\mathbf{x}) = \mathbf{A}\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^3$.

(b) Find a basis for the *range* of T

(c) Find a basis for the *kernel* of T.

4. Compute the following determinants by the indicated method

(a) $\det \begin{pmatrix} 3 & 3 & 0 & 2 \\ 0 & 2 & 0 & 1 \\ 1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ via row reduction

(b) $\det \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix}$ via a cofactor expansion

5. For each of the matrices $\mathbf{A}$ below

- Find the eigenvalues of $\mathbf{A}$
- Find the eigenvectors of $\mathbf{A}$
- Determine the both algebraic muliplicites and geometric multiplicity of each eigenvalue

(a) $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

(b) $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$