Math 3013
SAMPLE FIRST EXAM

1. Let

$$\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 3 & 1 \end{bmatrix} \quad , \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

Compute the matrix product **BA**

2. For each of the following augmented matrices, indicate

- the number of equations and the number of variables in the corresponding linear system
- whether or not the corresponding linear system has a solution
- if the corresponding linear system does have a solution, the number of free variables in the solution.

Hint: note that these augmented matrices are already in row echelon form.

(a) $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

(b) $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 1 \\ 0 & 2 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

3. (10 pts) Consider the following linear system

$$\begin{array}{rcr} x_1 - x_2 + 2x_3 & = & 1 \\ 2x_1 + x_2 + x_3 & = & -1 \\ -x_1 + x_2 + 2x_3 & = & 3 \end{array}$$

Write down the corresponding augmented matrix and row reduce it to row-echelon form.

4. Row reduce the following augmented matrix to **reduced** row-echelon form.

$$[\mathbf{A} \mid \mathbf{b}] = \left[\begin{array}{ccccc|c} 2 & 2 & 4 & 6 & 2 & 2 \\ 0 & 0 & 3 & 6 & 6 & 3 \\ 0 & 0 & 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

5. Suppose the augmented matrix below is the Reduced Row Echelon Form of an augmented matrix of a linear system. Display the solution of the linear system as a hyperplane (within the space of variables).

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & -2 & 1 & 1 \\ 0 & 0 & 1 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

6. (10 pts) Compute the inverse of

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

7. Write down precise definitions for the following terms

(a) **Subspace of** $\mathbb{R}^n$:

(b) **Basis of a Subspace**:

(c) **Linearly Independent Set of Vectors**:

8. Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 \end{pmatrix}$$

Find bases for

(a) the row space, $RowSp\,(\mathbf{A})$, of $\mathbf{A}$

(b) the column space, $ColSp\,(\mathbf{A})$, of $\mathbf{A}$

(c) the null space, $NullSp\,(\mathbf{A})$, of $\mathbf{A}$