Math 3013.004

FIRST EXAM

10:30 – 11:45 am, Feb. 23, 1999

Name:________________________________

1. (5 pts) Compute the angle between the vectors $\mathbf{u} = (1, 2, -1, 0)$ and $\mathbf{v} = (1, 1, 2, 1)$.

2. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \quad , \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 3 & 1 \end{bmatrix}$$

Compute the following matrices.

(a) (5 pts) $\sqrt{2}\mathbf{A}$

(b) (5 pts) $\mathbf{AB}$

(c) (5 pts) $\mathbf{BA}$

(d) (5 pts) $\mathbf{A} + 2\mathbf{B}$

3. Consider the following linear system

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 1 \\ 2x_1 + x_2 + x_3 &= -1 \\ 3x_1 + x_2 + 2x_3 &= 3 \\ x_1 + x_3 &= 4 \end{aligned}$$

(a) (5 pts) Write down the corresponding augmented matrix and reduce it to row-echelon form.

(b) (5 pts) Reduce the augmented matrix further to **reduced** row-echelon form.

(c) (5 pts) Write down the solution of the original linear system.

4. (10 pts) Compute the inverse of

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

and verify that you have the correct inverse.

5. (10 pts) Use the fact that

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & -1 \\ -2 & 3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \quad \Rightarrow \quad \mathbf{A}^{-1} = \begin{bmatrix} 0 & 1 & 3 \\ 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

to solve

$$\begin{bmatrix} 1 & -1 & -1 \\ -2 & 3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

6. (10 pts) Determine if the set $W = \{(x, y, z) \in \mathbb{R}^3 \mid y = x, z = 2x\}$ is a subspace of $\mathbb{R}^3$.

7. (10 pts) Find a basis for the solution set of the following homogeneous linear system.

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 0 \\ x_1 + x_2 + 3x_3 &= 0 \\ x_2 - 2x_3 &= 0 \end{aligned}$$

8. (20 pts) Mark each ot the following statements True or False. (Think carefully.)

____ (a) If $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are invertible $n \times n$ matrices, then $\mathbf{AC} = \mathbf{BC}$ implies $\mathbf{A} = \mathbf{B}$.
____ (b) If $\mathbf{A}$ and $\mathbf{B}$ are invertible $n \times n$ matrices, then $\mathbf{AB} = \mathbf{BA}$ implies $\mathbf{B} = \mathbf{A}^{-1}$.
____ (c) If a consistent linear system has more equations than unknowns, then there will be a unique solution.
____ (d) If a square linear system $\mathbf{Ax} = \mathbf{0}$ has only the trivial solution, then every linear system of the form $\mathbf{Ax} = \mathbf{b}$ will have a unique solution.
____ (e) If $\mathbf{x}_1$ and $\mathbf{x}_2$ are solutions of a consistent linear system $\mathbf{Ax} = \mathbf{b}$, then so is $\mathbf{x}_1 + \mathbf{x}_2$.
____ (f) If $\mathbf{p}$ is a solution of $\mathbf{Ax} = \mathbf{b}$ then every other solution can be written as $\mathbf{x} = \mathbf{p} + \mathbf{h}$ where $\mathbf{h}$ is a solution of the corresponding homogeneous equation.
____ (g) Every line in $\mathbb{R}^2$ is a subspace of $\mathbb{R}^2$.
____ (h) If $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are vectors in $\mathbf{R}^3$, then every vector in span$(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ can be represented as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.
____ (i) If every vector in a subspace W of $\mathbb{R}^4$ can be represented as a linear combination of vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \mathbf{R}^4$, then $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ form a basis for W.
____ (j) A square linear system $\mathbf{Ax} = \mathbf{b}$ has a unique solution if and only if $\mathbf{A}$ is row-equivalent to the identity matrix.