1. (5 pts) Compute the angle between the vectors \( \mathbf{u} = (1, 2, -1, 0) \) and \( \mathbf{v} = (1, 1, 2, 1) \).

2. Let

\[
\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 3 & 1 \end{bmatrix}
\]

Compute the following matrices.

(a) (5 pts) \( \sqrt{2} \mathbf{A} \)

(b) (5 pts) \( \mathbf{A} \mathbf{B} \)

(c) (5 pts) \( \mathbf{B} \mathbf{A} \)

(d) (5 pts) \( \mathbf{A} + 2\mathbf{B} \)
3. Consider the following linear system

\[
\begin{align*}
    x_1 - x_2 + 2x_3 &= 1 \\
    2x_1 + x_2 + x_3 &= -1 \\
    3x_1 + x_2 + 2x_3 &= 3 \\
    x_1 + x_3 &= 4
\end{align*}
\]

(a) (5 pts) Write down the corresponding augmented matrix and reduce it to row-echelon form.

(b) (5 pts) Reduce the augmented matrix further to \textbf{reduced} row-echelon form.

(c) (5 pts) Write down the solution of the original linear system.
4. (10 pts) Compute the inverse of 

\[ A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix} \]

and verify that you have the correct inverse.

5. (10 pts) Use the fact that 

\[ A = \begin{bmatrix} 1 & -1 & -1 \\ -2 & 3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 0 & 1 & 3 \\ 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix} \]

to solve 

\[ \begin{bmatrix} 1 & -1 & -1 \\ -2 & 3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \]
6. (10 pts) Determine if the set \( W = \{(x, y, z) \in \mathbb{R}^3 \mid y = x, z = 2x\} \) is a subspace of \( \mathbb{R}^3 \).

7. (10 pts) Find a basis for the solution set of the following homogeneous linear system.

\[
\begin{align*}
x_1 + 2x_2 + x_3 &= 0 \\
x_1 + x_2 + 3x_3 &= 0 \\
x_2 - 2x_3 &= 0
\end{align*}
\]

8. (20 pts) Mark each of the following statements True or False. (Think carefully.)

(a) If \( A, B \) and \( C \) are invertible \( n \times n \) matrices, then \( AC = BC \) implies \( A = B \).
(b) If \( A \) and \( B \) are invertible \( n \times n \) matrices, then \( AB = BA \) implies \( B = A^{-1} \).
(c) If a consistent linear system has more equations than unknowns, then there will be a unique solution.
(d) If a square linear system \( Ax = 0 \) has only the trivial solution, then every linear system of the form \( Ax = b \) will have a unique solution.
(e) If \( x_1 \) and \( x_2 \) are solutions of a consistent linear system \( Ax = b \), then so is \( x_1 + x_2 \).
(f) If \( p \) is a solution of \( Ax = b \) then every other solution can be written as \( x = p + h \) where \( h \) is a solution of the corresponding homogeneous equation.
(g) Every line in \( \mathbb{R}^2 \) is a subspace of \( \mathbb{R}^2 \).
(h) If \( v_1, v_2, \) and \( v_3 \) are vectors in \( \mathbb{R}^3 \), then every vector in \( \text{span}(v_1, v_2, v_3) \) can be represented as a linear combination of \( v_1, v_2, \) and \( v_3 \).
(i) If every vector in a subspace \( W \) of \( \mathbb{R}^4 \) can be represented as a linear combination of vectors \( v_1, v_2, v_3 \in \mathbb{R}^4 \), then \( v_1, v_2, \) and \( v_3 \) form a basis for \( W \).
(j) A square linear system \( Ax = b \) has a unique solution if and only if \( A \) is row-equivalent to the identity matrix.