Math 3013
Problem Set 9

1. Let $B_0 = \{[1,0,0], [0,1,0], [0,0,1]\}$ be the standard basis for $\mathbb{R}^3$ and let $B_1 = \{[1,1,1], [1,-1,1], [1,1,-1]\}$ be another basis for $\mathbb{R}^3$.

(a) What is the change of basis matrix $\mathbf{C}_{B_1 \to B_0}$ that connects coordinate vectors with respect to B_1 to coordinate vectors with respect to B_0?

(b) Suppose a vector $\mathbf{v}$ has coordinate vector $\mathbf{v}_{B_1} = [1,2,1]$ with respect to the basis B_1. What are its coordinates with respect to the standard basis B_0.

(c) What is the change of basis matrix $\mathbf{C}_{B_0 \to B_1}$ that connects standard coordinate vectors (with respect to B_0) to coordinate vectors with respect to B_1.

(d) If $\mathbf{v}_{B_0} = [2,1,2]$ what are its coordinates with respect to the basis B_1.

2. Suppose $B_1 = \{[1,1], [1,-1]\}$ and $B_2 = \{[2,1], [1,0]\}$. Find the change of basis matrix $\mathbf{C}_{B_1 \to B_2}$ that maps coordinate vectors with respect the basis B_1 to coordinate vectors with respect to the basis B_2.

3. Let $\mathbf{a} = [1,-1,1]$ and $\mathbf{b} = [2,1,1]$.

(a) Find the orthogonal projection of $\mathbf{a}$ onto $\mathbf{b}$.

(b) Find the component of $\mathbf{a}$ that is perpendicular to $\mathbf{b}$.

4. Let $\mathbf{v}_1 = [1,1,1]$, $\mathbf{v}_2 = [1,1,0]$, $\mathbf{v}_3 = [1,0,0]$. Given that these vectors are linearly independent, apply the Gram-Schmidt orthogonalization process to obtain a corresponding orthogonal basis for $\mathbb{R}^3$.

5. Let $\mathbf{v}_1 = [1,1,0]$ and $\mathbf{v}_2 = [3,4,2]$.

(a) Apply the Gram-Schmidt process to $\{\mathbf{v}_1, \mathbf{v}_2\}$ to an orthogonal basis for the subspace $\mathit{span}(\mathbf{v}_1, \mathbf{v}_2)$ of $\mathbb{R}^3$.

(b) Extend the orthogonal basis found in part (a) to an orthonormal basis for $\mathbb{R}^3$.

(c) Find the orthogonal projection of $[1,2,1]$ onto $W = \mathit{span}(\mathbf{v}_1, \mathbf{v}_2)$

6. Let $\mathbf{x}_1 = [1,-1,1,1]$, $\mathbf{x}_2 = [1,-1,1,-1]$.

(a) Apply the Gram-Schmidt process to $\{\mathbf{v}_1, \mathbf{v}_2\}$ to an orthogonal basis for the subspace $\mathit{span}(\mathbf{v}_1, \mathbf{v}_2)$ of $\mathbb{R}^4$.

(b) Extend the orthogonal basis found in part (a) to an orthonormal basis for $\mathbb{R}^4$.

(c) Find the orthogonal projection of $[2,1,1,1]$ onto $\mathit{span}(\mathbf{v}_1, \mathbf{v}_2)$

7. (a) Find an orthogonal basis for $\mathbb{R}^3$ that contains the vector $\mathbf{v} = [3,1,5]$.

(b) Find a basis for the orthogonal complement of $\mathbf{v}$ in $\mathbb{R}^3$.