

Math 3013
Problem Set 8

1. Find the characteristic polynomial, the real eigenvalues, and the corresponding eigenvectors for the following matrices.

$$(a) \mathbf{A} = \begin{bmatrix} 7 & 5 \\ -10 & -8 \end{bmatrix}$$

$$(b) \mathbf{A} = \begin{bmatrix} -7 & -5 \\ 16 & 17 \end{bmatrix}$$

$$(c) \mathbf{A} = \begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix}$$

$$(d) \mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ -4 & 2 & -1 \\ 4 & 0 & 3 \end{bmatrix}$$

$$(e) \mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ -8 & 4 & -5 \\ 8 & 0 & 9 \end{bmatrix}$$

$$(f) \mathbf{A} = \begin{bmatrix} -4 & 0 & 0 \\ -7 & 2 & -1 \\ 7 & 0 & 3 \end{bmatrix}$$

2. Find the eigenvalues λ_i and the corresponding eigenvectors \mathbf{v}_i for the following linear transformations.

$$(a) T([x, y]) = [2x - 3y, -3x + 2y]$$

$$(b) T([x_1, x_2, x_3]) = [x_1 + x_3, x_2, x_1 + x_3]$$

3. Find the eigenvalues λ_i , the corresponding eigenvectors \mathbf{v}_i of the following matrices. Also find an invertible matrix \mathbf{C} and a diagonal matrix \mathbf{D} such that $\mathbf{D} = \mathbf{C}^{-1}\mathbf{A}\mathbf{C}$.

$$(a) \mathbf{A} = \begin{bmatrix} -3 & 4 \\ 4 & 3 \end{bmatrix}$$

$$(b) \mathbf{A} = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$(c) \mathbf{A} = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix}$$

$$(d) \mathbf{A} = \begin{bmatrix} 6 & 3 & -3 \\ -2 & -1 & 2 \\ 16 & 8 & -7 \end{bmatrix}$$

$$(e) \mathbf{A} = \begin{bmatrix} -3 & 10 & -6 \\ 0 & 7 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Determine whether or not the following matrices are diagonalizable.

$$(a) \mathbf{A} = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 0 & -4 \\ 6 & -4 & 3 \end{bmatrix}$$

$$(b) \mathbf{A} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$