1. Show by direct calculation that

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = - \begin{vmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

(This demonstrates how the determinant behaves after a row interchange).

2. Compute the determinants of the following matrices.

(a) $\mathbf{A} = \begin{bmatrix} 2 & 3 & -1 \\ 5 & -7 & 1 \\ -3 & 2 & -1 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 5 & 2 & 4 & 0 \\ 2 & -3 & -1 & 2 \\ 3 & -4 & 3 & 7 \\ 1 & -1 & 0 & 1 \end{bmatrix}$

(c) $\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 3 & -1 & 2 & 0 & 0 \\ 0 & 4 & 1 & -1 & 2 \\ 0 & 0 & -3 & 2 & 4 \\ 0 & 0 & 0 & -1 & 3 \end{bmatrix}$

(d) $\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 2 & 0 & -3 \\ 0 & -2 & 1 & 0 & 0 \\ 5 & -3 & 2 & 0 & 0 \\ -3 & 4 & 0 & 0 & 0 \end{bmatrix}$

(e) $\mathbf{A} = \begin{bmatrix} 2 & -1 & 3 & 0 & 0 \\ 0 & 1 & 4 & 0 & 0 \\ -5 & 2 & 6 & 0 & 0 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -2 & 8 \end{bmatrix}$

3. For each matrix $\mathbf{A}$ below, let $\mathbf{C}_A$ be the cofactor matrix $(\mathbf{C}_A)_{ij} = (-1)^{i+j} \det \left(A^{(i,j)} \right)$. Use the formula $\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} (\mathbf{C}_A)^T$ to compute $\mathbf{A}^{-1}$.

(a) $\mathbf{A} = \begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 3 & 0 & 4 \\ -2 & 1 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

4. Solve the following systems of linear equations using Cramers's Rule.

(a) $\begin{array}{rcl} x_1 - 2x_2 & = & 1 \\ 3x_1 + 4x_2 & = & 3 \end{array}$

(b) $\begin{array}{rcl} x_1 + 2x_2 - x_3 & = & -2 \\ 2x_1 + x_2 + x_3 & = & 0 \\ 3x_1 - x_2 + 5x_3 & = & 1 \end{array}$