Math 3013
Problem Set 6

1. Compute the determinants of the following matrices.

(a) $\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}$

2. Use a cofactor expansion to compute the determinat of $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ -1 & 1 & 3 & 0 \\ 2 & 3 & 1 & 4 \end{bmatrix}$

3. Use row reduction to compute the determinant of $\mathbf{A} = \begin{bmatrix} 1 & 1 & -1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & -1 & 3 \\ 2 & 1 & 1 & 2 \end{bmatrix}$

4. Use the result of Problem 1(b) to determine if the vectors $\mathbf{a} = [1, 0, -1]$, $\mathbf{b} = [3, 2, 1]$ and $\mathbf{c} = [-1, 1, 0]$ are linearly independent.

5. Use the result of Problem 3 to determine if the linear system

$$\begin{aligned} x_1 + x_2 - x_3 + 2x_4 &= 0 \\ 2x_1 + x_2 + x_4 &= 0 \\ 3x_1 + 2x_2 - x_3 + 3x_4 &= 0 \\ 2x_1 + x_2 + x_3 + 2x_4 &= 0 \end{aligned}$$

has a unique solution.

6. Use Crammer's Rule to determine the solution (if any) of

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 2 \\ 2x_1 - 2x_2 &= 4 \\ x_2 + x_3 &= 3 \end{aligned}$$

7. Compute the cofactor $\mathbf{C}$ of matrix of $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and use the fomula

$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \mathbf{C}^T$$

to get a general formula for the inverse of a 2×2 matrix.