Math 3013
Problem Set 4

1. Determine whether the indicated subset is a subspace of the given $\mathbb{R}^n$.

(a) $W = \{[r, -r] \mid r \in \mathbb{R}\}$ in $\mathbb{R}^2$

(b) $W = \{[n, m] \mid \ n$ and n are integers$\}$ in $\mathbb{R}^2$

(c) $W = \{[x, y, z] \mid x, y, z \in \mathbb{R}$ and $z = 3x + 2\}$ in $\mathbb{R}^3$

(d) $W = \{[x, y, z] \mid x, y, z \in \mathbb{R}$ and $z = 1,\ y = 2x\}$ in $\mathbb{R}^3$

(e) $W = \{[2x_1, 3x_2, 4x_3, 5x_4] \mid x_i \in \mathbb{R}\}$ in $\mathbb{R}^4$

2. Prove that the line $y = mx$ is a subspace of $\mathbb{R}^2$. (Hint: write the line as $W = \{[x, mx] \mid x \in \mathbb{R}\}$.)

3. Find a basis for the solution set of the following homogeneous linear system.

$$\begin{aligned} 3x_1 + x_2 + x_3 &= 0 \\ 6x_1 + 2x_2 + 2x_3 &= 0 \\ -9x_1 - 3x_2 - 3x_3 &= 0 \end{aligned}$$

4. Give a geometric criterion for a set of two vectors in $\mathbb{R}^n$ to be linearly independent.

5. Find a basis for the row space of the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 0 & 4 & 2 \\ 3 & 2 & 8 & 7 \end{bmatrix}$$

6. Find a basis for the column space of the matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 3 & 1 \\ 5 & 2 & 1 \\ 1 & 7 & 2 \\ 6 & -2 & 0 \end{bmatrix}$$

7. Find a basis for the subspace of $\mathbb{R}^4$ that is spanned by the vectors $[1, 2, 1, 1]$, $[2, 1, 0, -1]$, $[-1, 4, 3, 8]$, and $[0, 3, 2, 5]$.

8. Determine whether the following sets of vectors are dependent or independent.

(a) $\{[1, 3], [-2, -6]\}$ in $\mathbb{R}^2$.

(b) $\{[1, -4, 3], [3, -11, 2], [1, -3, -4]\}$ in $\mathbb{R}^3$.

9. For each of the following matrices find the rank of the matrix, a basis for its row space, a basis for its column space, and a basis for its null space.

(a)

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & -3 & 1 \\ 3 & 4 & 2 & 2 \end{bmatrix}$$

(b)

$$\mathbf{A} = \begin{bmatrix} 0 & 6 & 6 & 3 \\ 1 & 2 & 1 & 1 \\ 4 & 1 & -3 & 4 \\ 1 & 3 & 2 & 0 \end{bmatrix}$$

(c)

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & 1 & 0 & 2 \\ 0 & 2 & 1 & 1 \end{bmatrix}$$