Math 3013
Problem Set 3

1. Find the inverses of the following matrices. If a matrix inverse exists, also express it as a product of elementary matrices.

(a) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 6 \\ 4 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 5 \\ 0 & -1 & 1 \end{bmatrix}$

2. Find the inverse of $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}$

3. Determine if the following matrix is invertible.

$$\begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & -1 & -3 & 4 \\ 1 & 0 & -1 & 2 \\ -3 & 0 & 0 & -1 \end{bmatrix}$$

4. Show that the following matrix is invertible and find its inverse.

$$\mathbf{A} = \begin{bmatrix} 2 & -3 \\ 5 & -7 \end{bmatrix}$$

5. Let

$$\mathbf{A}^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix}$$

If possible, find a matrix $\mathbf{C}$ such that

$$\mathbf{AC} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 4 & 1 \end{bmatrix}$$