

Math 3013  
Problem Set 2

1. Reduce the following matrices to row-echelon form, and then to reduced row-echelon form.

(a)

$$\begin{bmatrix} 2 & 1 & 4 \\ 1 & 3 & 2 \\ 3 & -1 & 6 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 0 & 2 & -1 & 3 \\ -1 & 1 & 2 & 0 \\ 1 & 1 & -3 & 3 \\ 1 & 5 & 5 & 9 \end{bmatrix}$$

(c)

$$\begin{bmatrix} -1 & 3 & 0 & 1 & 4 \\ 1 & -3 & 0 & 0 & -1 \\ 2 & -6 & 2 & 4 & 0 \\ 0 & 0 & 1 & 3 & -4 \end{bmatrix}$$

2. For each linear system below, construct the corresponding augmented matrix

(a)

$$\begin{aligned} x_1 + 2x_3 &= 1 \\ x_2 + x_3 + 3x_4 &= -2 \end{aligned}$$

(b)

$$\begin{aligned} x_5 &= 3 \\ x_1 + x_3 + x_4 &= 2 \\ x_2 - x_5 &= 1 \end{aligned}$$

3. The augmented matrices below correspond to linear systems. Describe the nature of the solutions of the corresponding linear system, and then, if possible, present the solution as a set of vectors. (Hint: note that the augmented matrices are already in reduced row echelon form).

(a)  $\left[ \begin{array}{cccc|c} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$

(b)  $\left[ \begin{array}{ccccc|c} 1 & -1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$

4. Find all solutions of the given linear system.

(a)

$$\begin{aligned} 2x - y &= 8 \\ 6x - 5y &= 32 \end{aligned}$$

(b)

$$\begin{aligned} y + z &= 6 \\ 3x - y + z &= -7 \\ x + y - 3z &= -13 \end{aligned}$$

5. Determine whether the vector

$$\mathbf{b} = \begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix}$$

is in the span of the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -3 \\ -1 \\ 5 \end{bmatrix}$$