Math 3013
Problem Set 1

1. Let $\mathbf{u} = [1, 2, 1, 0]$, $\mathbf{v} = [-2, 0, 1, 6]$ and $\mathbf{w} = [3, -5, 1, -2]$. Compute $\mathbf{u} - 2\mathbf{v} + 4\mathbf{w}$.

2. Find the vector which, when translated, represents geometrically an arrow reaching from the point $(-1, 3)$ to the point $(4, 2)$ in $\mathbb{R}^2$.

3. Let $\mathbf{u} = [-1, 3, 4]$ and $\mathbf{v} = [2, 1, -1]$. Compute $\|-\mathbf{u}\|$ and $\|\mathbf{v} + \mathbf{u}\|$.

4. Compute the angle between $[1, -1, 2, 3, 0, 4]$ and $[7, 0, 1, 3, 2, 4]$ in $\mathbb{R}^6$.

5. Prove that $(2, 0, 4)$, $(4, 1, -1)$ and $(6, 7, 7)$ are the vertices of a right triangle in $\mathbb{R}^3$.

6. Specify the line that passes through both the points $(1, 0, 2)$ and $(2, 1, 0)$ as a set of vectors. (I.e., find vectors $\mathbf{p}_0$ and v such that the line corresponds to the set $\ell = \{\mathbf{p}_0 + t\mathbf{v} \mid t \in \mathbb{R}\}$.)

7. Let

$$\mathbf{A} = \begin{bmatrix} -2 & 1 & 3 \\ 4 & 0 & -1 \end{bmatrix} \quad , \quad \mathbf{B} = \begin{bmatrix} 4 & 1 & -2 \\ 5 & -1 & 3 \end{bmatrix} \quad , \quad \mathbf{C} = \begin{bmatrix} 2 & -1 \\ 0 & 6 \\ -3 & 2 \end{bmatrix} \quad , \quad \mathbf{D} = \begin{bmatrix} -4 & 2 \\ 3 & 5 \\ -1 & -3 \end{bmatrix}$$

(a) $3\mathbf{A}$

(b) $\mathbf{A} + \mathbf{B}$

(c) $\mathbf{AB}$

(d) $\mathbf{A}^2$

(e) $(2\mathbf{A} - \mathbf{B})\mathbf{D}$

(f) $\mathbf{ADB}$

8. Consider the row and column vectors

$$\mathbf{x} = [-2, 3, -1] \quad , \quad \mathbf{y} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$$

Compute the matrix products $\mathbf{xy}$ and $\mathbf{yx}$.