Math 3013.004

FINAL EXAM

1:00 – 2:50 pm, May 3, 1999

Name:______________________________

1. Consider the following linear system

$$\begin{aligned} x_1 + x_2 + x_3 - x_4 &= 6 \\ x_2 + x_3 &= 5 \\ 2x_1 - x_2 + x_3 - 2x_4 &= 3 \\ -x_1 + x_3 + x_4 &= 2 \end{aligned}$$

(a) (5 pts) Write down the corresponding augmented matrix and reduce it to row-echelon form.

- $$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 6 \\ 0 & 1 & 1 & 0 & 5 \\ 2 & -1 & 1 & -2 & 3 \\ -1 & 0 & 1 & 1 & 2 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 6 \\ 0 & 1 & 1 & 0 & 5 \\ 0 & -3 & -1 & 0 & -9 \\ 0 & 1 & 2 & 0 & 8 \end{array}\right]$$
$$\longrightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 6 \\ 0 & 1 & 1 & 0 & 5 \\ 0 & 0 & 2 & 0 & 6 \\ 0 & 0 & 1 & 0 & 3 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 6 \\ 0 & 1 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\square$

(b) (5 pts) Reduce the augmented matrix further to **reduced** row-echelon form.

- $$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 6 \\ 0 & 1 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\square$

(c) (5 pts) Use the result of (b) to find the solution of the original linear system.

- $$\begin{array}{c} x_1 - x_4 = 1 \\ x_2 = 2 \\ x_3 = 3 \\ 0 = 0 \end{array} \quad \Rightarrow \quad \begin{array}{c} x_1 = 1 + r \\ x_2 = 2 \\ x_3 = 3 \\ x_4 = r \end{array} \quad \text{where } r \text{ is an arbitary real number}$$

$\square$

2. (10 pts) Compute the inverse of

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

and verify that you have the correct inverse.

- $$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -2 & -1 & -2 & 1 & 0 \\ 0 & -2 & 0 & -1 & 0 & 1 \end{array}\right]$$
$$\longrightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right]$$
$$\longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right] \Rightarrow \mathbf{A}^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ 1 & -1 & 1 \end{bmatrix}$$
One verifies that
$$\mathbf{A}^{-1}\mathbf{A} = \begin{bmatrix} -1 & 1 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

□

3. (10 pts) Find a basis for the solution set of the following homogeneous linear system.

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 0 \\ x_1 + x_2 + 3x_3 &= 0 \\ x_2 - 2x_3 &= 0 \end{aligned}$$

- The sollution set is the null space of the following row-equivalent matrices:
$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 3 \\ 0 & 1 & -2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 0 & 1 & -2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$
The null space of the last matrix is given by
$$\begin{aligned} x_1 + 5x_3 &= 0 \\ x_2 - 2x_3 &= 0 \\ 0 &= 0 \end{aligned} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} -5x_3 \\ 2x_3 \\ x_3 \end{bmatrix} \in \mathit{span}\left(\begin{bmatrix} -5 \\ 2 \\ 1 \end{bmatrix} \right)$$

□

4.(5 pts) Determine if $S = \{[x, x+1, y] \mid x, y \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$.

- Consider an arbitary $[x, x+1, y]$ element of S. If S is a subspace it must be closed under scalar multiplication. However, if $\lambda \neq 1$ then

$$\lambda [x, x+1, y] = [\lambda x, \lambda x + \lambda, \lambda y]$$

Note that the second component of the vector on the right is **not** 1 plus the first component. Therefore, it is not in S. Hence, S is not closed under scalar multiplication; and so S is not a subspace. □

5. Consider the following matrix: $\mathbf{A} = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 3 & 2 \end{bmatrix}$

(a) (5 pts) Find a basis for the column space of $\mathbf{A}$.

- Row reducing $\mathbf{A}$ yields

$$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 3 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \mathbf{A}'$$

The pivots of the row-echelon form $\mathbf{A}'$ of $\mathbf{A}$ are in the first and second columns, therefore the first two columns of $\mathbf{A}$ form a basis for the column space of $\mathbf{A}$.

$$ColSp(\mathbf{A}) = span\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}\right)$$

□

(b) (5 pts) Find a basis for the row space of $\mathbf{A}$.

The non-zero rows of the row-echelon form $\mathbf{A}'$ of $\mathbf{A}$ form a basis for the row space of $\mathbf{A}$. Thus,

$$RowSp(\mathbf{A}) = span([1, 2, 2, 1], [0, 1, -1, -1])$$

- □

(c) (5 pts) Find a basis for the null space of $\mathbf{A}$.

If we complete the row reduction of $\mathbf{A}$ (to reduced row-echelon form) we obtain

$$\begin{bmatrix} 1 & 0 & 4 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The solution space to the corresponding linear system is

$$\begin{array}{c} x_1 = -4x_3 - 3x_4 \\ x_2 = x_3 + x_4 \end{array} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} -4x_3 - 3x_4 \\ x_3 + x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -4 \\ 1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix} \in span\left(\begin{bmatrix} -4 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix}\right)$$

- □

6. Consider the following mapping: $T : \mathbb{R}^2 \to \mathbb{R}^3 : T([x_1, x_2]) = [x_1 + x_2, x_1 + 2x_2, x_1 - x_2]$

(a) (5 pts) Show that T is a linear transformation.

- Let $\mathbf{x} = [x_1, x_2]$. Then

$$T(\lambda\mathbf{x}) = T([\lambda x_1, \lambda x_2]) = [\lambda x_1 + \lambda x_2, \lambda x_1 + 2\lambda x_2, \lambda x_1 - \lambda x_2] = \lambda [x_1 + x_2, x_1 + 2x_2, x_1 - x_2] = \lambda T(\mathbf{x})$$

So T preserves scalar multiplication. Let $\mathbf{x}' = [x_1', x_2']$. Then

$$\begin{aligned} T(\mathbf{x} + \mathbf{x}') &= T(x_1 + x_1', s_2 + x_2') = [x_1 + x_1 + x_2 + x_2', x_1 + x_1' + 2(x_2 + x_2'), x_1 + x_1 - (x_2 + x_2')] \\ &= [x_1 + x_2, x_1 + 2x_2, x_1 - x_2] + [x_1' + x_2', x_1' + 2x_2', x_1' - x_2'] \\ &= T(\mathbf{x}) + T(\mathbf{x}') \end{aligned}$$

So T also preserves vector addition. Since T preserves both scalar multiplication and vector addition it is a linear transformation. $\square$

(b) (5 pts) Find the matrix that represents T.

- $$\begin{array}{c} T([1,0]) = [1,1,1] \\ T([0,1]) = [1,2,-1] \end{array} \quad \Rightarrow \quad \mathbf{A}_T = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & -1 \end{bmatrix}$$ $\square$

- $\square$

(c) (5 pts) Find a basis for the range of T.

- The range of T will coincide with the column space of $\mathbf{A}_T$. Since the two columns of $\mathbf{A}_T$ are not constant multiples of one another, they are linearly independent, and hence form a basis for the column space of $\mathbf{A}$. Thus,

$$ColSp(\mathbf{A}) = span\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}\right)$$

$\square$

7. (5 pts) Let $p_1 = 1 - 2x + x^2$, $p_2 = 1 - x^2$, $p_3 = 2 - 3x + x^2$.. Find a basis for $span(p_1, p_2, p_3)$.

- Using the standard translation of polynomials into numerical vectors we have

$$\begin{array}{c} 1 \leftrightarrow [1,0,0] \\ x \leftrightarrow [0,1,0] \\ x^2 \leftrightarrow [0,0,1] \end{array} \quad \Rightarrow \quad \begin{array}{c} p_1 \to [1,-2,1] = \mathbf{v}_1 \\ p_2 \to [1,0,-1] = \mathbf{v}_2 \\ p_3 \to [2,-3,1] = \mathbf{v}_3 \end{array}$$

We now find a basis for $span(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

$$[\mathbf{v}_1 \mid \mathbf{v}_2 \mid \mathbf{v}_3] = \mathbf{A} = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & -3 \\ 1 & -1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 0 & -2 & -1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

The pivots of last matrix are in the first two columns; therefore, the first two columns of $\mathbf{A}$ are a basis for the its column space. Hence, $\mathbf{v}_1$ and $\mathbf{v}_2$ are a basis for $span(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$. Hence, $p_1 = 1 - 2x + x^2$ and $p_2 = 1 - x^2$ are a basis for $span(p_1, p_2, p_3)$. $\square$

8. (5 pts) Compute the determnant of the following matrix.

$$\mathbf{A} = \begin{bmatrix} 2 & 3 & 4 & 6 \\ 2 & 0 & -9 & 6 \\ 4 & 1 & 0 & 2 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

- This computation will be simplest if we row reduce $\mathbf{A}$ a bit.

$$\det(\mathbf{A}) = \begin{vmatrix} 2 & 3 & 4 & 6 \\ 0 & 3 & -13 & 0 \\ 0 & -5 & -8 & -10 \\ 0 & 1 & -1 & 0 \end{vmatrix} = 2\begin{vmatrix} 3 & -13 & 0 \\ -5 & -8 & -10 \\ 1 & -1 & 0 \end{vmatrix} - 3\begin{vmatrix} 0 & -13 & 0 \\ 0 & -8 & -10 \\ 0 & -1 & 0 \end{vmatrix} + 4\begin{vmatrix} 0 & 3 & 0 \\ 0 & -5 & -10 \\ 0 & 1 & -1 \end{vmatrix} - 6\begin{vmatrix} 0 & 3 & -13 \\ 0 & -5 & -8 \\ 0 & 1 & -1 \end{vmatrix}$$

The last three determinants on the right vanish because the first columns are filled with zeros. Hence,

$$\begin{aligned} \det(\mathbf{A}) &= 2\begin{vmatrix} 3 & -13 & 0 \\ -5 & -8 & -10 \\ 1 & -1 & 0 \end{vmatrix} = 2\left(3\begin{vmatrix} -8 & -10 \\ -1 & 0 \end{vmatrix} - (-13)\begin{vmatrix} -5 & -10 \\ 1 & 0 \end{vmatrix} + 0\begin{vmatrix} -5 & -8 \\ 1 & -1 \end{vmatrix}\right) \\ &= (2)(3)(-10) + 2(13)(10) = 200 \end{aligned}$$

□

9. (5 pts) Find the characteristic polynomial of the following matrix.

$$\mathbf{A} = \begin{bmatrix} -2 & 0 & -1 \\ 0 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix}$$

-

$$\begin{aligned} P_{\mathbf{A}}(\lambda) &= \det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} -2-\lambda & 0 & -1 \\ 0 & 2-\lambda & 0 \\ 3 & 0 & 2-\lambda \end{vmatrix} = (-2-\lambda)\begin{vmatrix} 2-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ 3 & 2-\lambda \end{vmatrix} + (-1)\begin{vmatrix} 0 & 2-\lambda \\ 3 & 0 \end{vmatrix} \\ &= (-2-\lambda)(2-\lambda)^2 + 3(2-\lambda) = -2 + \lambda + 2\lambda^2 - \lambda^3 = -(\lambda-1)(\lambda-2)(\lambda+1) \end{aligned}$$

□

10. (10 pts) Given that $\det(\mathbf{A} - \lambda\mathbf{I}) = -(\lambda-1)^2(\lambda-2)$, find the eigenvalues and eigenvectors of

$$\mathbf{A} = \begin{bmatrix} 3 & -1 & 0 \\ 3 & -1 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

- The eigenvalues are the roots of the characteristic polynomial $\det(\mathbf{A} - \lambda\mathbf{I})$; i.e, $\lambda = 1, 2$. The eigenspace corresponding to $\lambda = 1$ is the null space of

$$\mathbf{A} - (1)\mathbf{I} = \begin{bmatrix} 2 & -1 & 0 \\ 3 & -2 & 1 \\ 1 & -1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 1 & -1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & -1 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

or the solution space of

$$\begin{matrix} x_1 - x_3 = 0 \\ x_2 - 2x_3 = 0 \\ 0 = 0 \end{matrix} \quad \Rightarrow \quad \mathbf{v}_1 = \begin{bmatrix} x_3 \\ 2x_3 \\ x_3 \end{bmatrix} \in span\left(\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}\right)$$

The eigenspace corresponding to the root $\lambda = 2$ is the null space of the matrix

$$\mathbf{A} - (2)\mathbf{I} = \begin{bmatrix} 1 & -1 & 0 \\ 3 & -3 & 1 \\ 1 & -1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

or the solution set of

$$\begin{array}{c} x_1 - x_2 = 0 \\ x_3 = 0 \\ 0 = 0 \end{array} \qquad \Rightarrow \qquad \mathbf{v}_2 = \begin{bmatrix} x_2 \\ x_2 \\ 0 \end{bmatrix} \in span\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right)$$

$\square$

11. (10 pts) Find the eigenvalues and eigenvectors of the following linear transformation: $T([x,y]) = [x+2y, 2x+y]$

- $$\mathbf{A}_T = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad \Rightarrow \quad \det(\mathbf{A}_T - \lambda \mathbf{I}) = (1-\lambda)(1-\lambda) - 4 = \lambda^2 - 2\lambda - 3 = (\lambda+1)(\lambda-3)$$

  So the eigenvalues of T are $\lambda = -1, 3$.

  The eigenspace corresponding to the eigenvalue -1 is the null space of $\mathbf{A}_T - (-1)\mathbf{I}$

  $$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad \Rightarrow \quad \begin{matrix} x_1 + x_2 = 0 \\ 0 = 0 \end{matrix} \quad \Rightarrow \quad \mathbf{v}_{-1} \in span\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix} \right)$$

  The eigenspace corresponding to the eigenvalue 3 is the null space of $\mathbf{A}_T - (3)\mathbf{I}$

  $$\begin{bmatrix} -2 & 2 \\ 2 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \quad \Rightarrow \quad \begin{matrix} x_1 - x_2 = 0 \\ 0 = 0 \end{matrix} \quad \Rightarrow \quad \mathbf{v}_{-1} \in span\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$$

  $\square$

12. (15 pts) Let $\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 0 & -3 \end{bmatrix}$. Find a diagonal matrix $\mathbf{D}$ and an invertible matrix $\mathbf{C}$, such that $\mathbf{D} = \mathbf{C}^{-1}\mathbf{A}\mathbf{C}$.

- $$\det(\mathbf{A} - \lambda \mathbf{I}) = (1-\lambda)(-3-\lambda) - 0 \quad \Rightarrow \quad \lambda = 1, -3$$

  The eigenspace corresponding the eigenvalue $\lambda = 1$ is the null space of $\mathbf{A} - (1)\mathbf{I}$

  $$\begin{bmatrix} 0 & 4 \\ 0 & -4 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \Rightarrow \quad \begin{matrix} x_2 = 0 \\ 0 = 0 \end{matrix} \quad \Rightarrow \quad \mathbf{v}_1 = \begin{bmatrix} x_1 \\ 0 \end{bmatrix} \in span\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$$

  The eigenspace corresponding the eigenvalue $\lambda = -3$ is the null space of $\mathbf{A} - (-3)\mathbf{I}$

  $$\begin{bmatrix} 4 & 4 \\ 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad \Rightarrow \quad \begin{matrix} x_1 + x_2 = 0 \\ 0 = 0 \end{matrix} \quad \Rightarrow \quad \mathbf{v}_2 = \begin{bmatrix} -x_2 \\ x_2 \end{bmatrix} \in span\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix} \right)$$

  So

  $$\mathbf{D} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix} \quad , \quad \mathbf{C} = [\mathbf{v}_1 \mid \mathbf{v}_2] = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

13. (30 pts) Mark each ot the following statements True or False. (Think carefully.)

__*T*__ (a) If $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are invertible $n \times n$ matrices, then $\mathbf{AC} = \mathbf{BC}$ implies $\mathbf{A} = \mathbf{B}$.
– ☐

__*F*__ (b) If $\mathbf{A}$ and $\mathbf{B}$ are invertible $n \times n$ matrices, then $\mathbf{AB} = \mathbf{BA}$ implies $\mathbf{B} = \mathbf{A}^{-1}$.
– ☐

__*F*__ (c) If a consistent linear system has more equations than unknowns, then there will be a unique solution.
– ☐

__*T*__ (d) If a square linear system $\mathbf{Ax} = \mathbf{0}$ has only the trivial solution, then every linear system of the form $\mathbf{Ax} = \mathbf{b}$ will have a unique solution.
– ☐

__*T*__ (e) If $\mathbf{p}$ is a solution of $\mathbf{Ax} = \mathbf{b}$ then every other solution can be written as $\mathbf{x} = \mathbf{p} + \mathbf{h}$ where $\mathbf{h}$ is a solution of the corresponding homogeneous equation.
– ☐

__*F*__ (f) Every line in $\mathbb{R}^2$ is a subspace of $\mathbb{R}^2$.
– ☐

__*F*__ (g) If every vector in a subspace W of $\mathbb{R}^4$ can be represented as a linear combination of vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \mathbf{R}^4$, then $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ form a basis for W.
– ☐

__*T*__ (h) A square linear system $\mathbf{Ax} = \mathbf{b}$ has a unique solution if and only if $\mathbf{A}$ is row-equivalent to the identity matrix.
– ☐

__*T*__ (i) The dimension of the row space of a matrix is the same as the dimension of the column space.
– ☐

__*T*__ (j) If $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is a set of vectors in $\mathbb{R}^n$ such every $\mathbf{v} \in \mathbb{R}^n$ can be expressed uniquely as a linear combination of the form $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_k\mathbf{v}_k$, then $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is a basis for $\mathbb{R}^n$.
– ☐

__*T*__ (k) Every $n \times n$ matrix has n not necessarily distinct and possibly complex eigenvectors.
– ☐

__*F*__ (l) There can be only one eigenvector associated with a given eigenvalue of a linear transformation.
– ☐

__*T*__ (m) There can be only one eigenvalue associated with a given eigenvector of a linear transformation.
– ☐

__*T*__ (n) If an $n \times n$ matrix is symmetric, then it is diagonalizable.
– ☐

__*F*__ (o) If the determinant of a matrix is not equal to zero then it is diagonalizable. ☐