Math 3013
Homework Set 9

Problems from §5.2 (pgs. 315-317 of text): 1,2,3,4,5,9,10,13

1. (Problems 5.2.1, 5.2.2, 5.2.3, 5.2.4, 5.2.5 in text) Find the eigenvalues λ_i, the corresponding eigenvectors $\mathbf{v}_i$ of the following matrices. Also find an invertible matrix $\mathbf{C}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{D} = \mathbf{C}^{-1}\mathbf{AC}$.

(a) $\mathbf{A} = \begin{bmatrix} -3 & 4 \\ 4 & 3 \end{bmatrix}$

- First, we calculate the eigenvalues and eigenvectors of $\mathbf{A}$.

$$0 = \det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} -3-\lambda & 4 \\ 4 & 3-\lambda \end{vmatrix} = \lambda^2 - 25 = (\lambda-5)(\lambda+5) \quad \Rightarrow \quad \lambda = 5, -5$$

The eigenspace corresponding to the eigenvalue $\lambda_1 = 5$ is the null space of

$$\mathbf{A} - (5)\mathbf{I} = \begin{bmatrix} -8 & 4 \\ 4 & -2 \end{bmatrix} \quad \rightsquigarrow \quad \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$$

or, equivalently, the solution space of

$$\begin{matrix} 2x_1 - x_2 = 0 \\ 0 = 0 \end{matrix} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} 2x_2 \\ x_2 \end{bmatrix} \in span\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right)$$

So the eigenspace corresponding to the eigenvalue $\lambda_1 = 5$ is the subspace generated by the vector

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

The eigenspace corresponding to the eigenvalue $\lambda_2 = -5$ is the null space of

$$\mathbf{A} - (-5)\mathbf{I} = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix} \quad \rightsquigarrow \quad \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

or, equivalently, the solution space of

$$\begin{matrix} x_1 + 2x_2 = 0 \\ 0 = 0 \end{matrix} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} -\frac{1}{2}x_2 \\ x_2 \end{bmatrix} \in span\left(\begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}\right) = span\left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$$

So the eigenspace corresponding to the eigenvalue $\lambda_2 = -5$ is the subspace generated by the vector

$$\mathbf{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Now that we know the eigenvalues and eigenvectors of $\mathbf{A}$, we can write down the diagonal matrix $\mathbf{D}$ by arranging the eigenvalues of $\mathbf{A}$ along the main diagonal of $\mathbf{D}$

$$\mathbf{D} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}$$

The matrix $\mathbf{C}$ can be written down by arranging the eigenvectors of $\mathbf{A}$ (in order) as the column vectors of a 2×2 matrix:

$$\mathbf{C} = [\mathbf{v}_1 \mid \mathbf{v}_2] = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$$

One can easily verify that

$$\mathbf{C}^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

and that $\mathbf{D} = \mathbf{C}^{-1}\mathbf{AC}$ (however, this fact is already guaranteed by the way we constructed the matrices $\mathbf{D}$ and $\mathbf{C}$).

(b) $\mathbf{A} = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

- First, we calculate the eigenvalues and eigenvectors of $\mathbf{A}$.

$$0 = \det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} 3-\lambda & 2 \\ 1 & 4-\lambda \end{vmatrix} = \lambda^2 - 7\lambda + 10 = (\lambda-2)(\lambda-5) \quad \Rightarrow \quad \lambda = 2, 5$$

The eigenspace corresponding to the eigenvalue $\lambda_1 = 2$ is the null space of

$$\mathbf{A} - (2)\mathbf{I} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \quad \rightsquigarrow \quad \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

or, equivalently, the solution space of

$$\begin{array}{l} x_1 + 2x_2 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} -2x_2 \\ x_2 \end{bmatrix} \in span\left(\begin{bmatrix} -2 \\ 1 \end{bmatrix}\right)$$

So the eigenspace corresponding to the eigenvalue $\lambda_1 = 2$ is the subspace generated by the vector

$$\mathbf{v}_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

The eigenspace corresponding to the eigenvalue $\lambda_2 = 5$ is the null space of

$$\mathbf{A} - (5)\mathbf{I} = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix} \quad \rightsquigarrow \quad \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

or, equivalently, the solution space of

$$\begin{array}{l} x_1 - x_2 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} x_2 \\ x_2 \end{bmatrix} \in span\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$$

So the eigenspace corresponding to the eigenvalue $\lambda_2 = 5$ is the subspace generated by the vector

$$\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Now that we know the eigenvalues and eigenvectors of $\mathbf{A}$, we can write down the diagonal matrix $\mathbf{D}$ by arranging the eigenvalues of $\mathbf{A}$ along the main diagonal of $\mathbf{D}$

$$\mathbf{D} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$$

The matrix $\mathbf{C}$ can be written down by arranging the eigenvectors of $\mathbf{A}$ (in order) as the column vectors of a 2×2 matrix:

$$\mathbf{C} = [\mathbf{v}_1 \mid \mathbf{v}_2] = \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}$$

(c) $\mathbf{A} = \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix}$

- First, we calculate the eigenvalues and eigenvectors of $\mathbf{A}$.

$$0 = \det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} 7-\lambda & 8 \\ -4 & -5-\lambda \end{vmatrix} = \lambda^2 - 2\lambda - 3 = (\lambda-3)(\lambda+1) \quad \Rightarrow \quad \lambda = 3, -1$$

The eigenspace corresponding to the eigenvalue $\lambda_1 = 3$ is the null space of

$$\mathbf{A} - (3)\mathbf{I} = \begin{bmatrix} 4 & 8 \\ -4 & -8 \end{bmatrix} \quad \rightsquigarrow \quad \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

or, equivalently, the solution space of

$$\begin{array}{l} x_1 + 2x_2 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} -2x_2 \\ x_2 \end{bmatrix} \in span\left(\begin{bmatrix} -2 \\ 1 \end{bmatrix}\right)$$

So the eigenspace corresponding to the eigenvalue $\lambda_1 = 3$ is the subspace generated by the vector

$$\mathbf{v}_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

The eigenspace corresponding to the eigenvalue $\lambda_2 = -1$ is the null space of

$$\mathbf{A} - (-1)\mathbf{I} = \begin{bmatrix} 8 & 8 \\ -4 & -4 \end{bmatrix} \quad \rightsquigarrow \quad \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

or, equivalently, the solution space of

$$\begin{array}{l} x_1 + x_2 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} -x_2 \\ x_2 \end{bmatrix} \in span\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix} \right)$$

So the eigenspace corresponding to the eigenvalue $\lambda_2 = -1$ is the subspace generated by the vector

$$\mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Now that we know the eigenvalues and eigenvectors of $\mathbf{A}$, we can write down the diagonal matrix $\mathbf{D}$ by arranging the eigenvalues of $\mathbf{A}$ along the main diagonal of $\mathbf{D}$

$$\mathbf{D} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$$

The matrix $\mathbf{C}$ can be written down by arranging the eigenvectors of $\mathbf{A}$ (in order) as the column vectors of a 2×2 matrix:

$$\mathbf{C} = [\mathbf{v}_1 \mid \mathbf{v}_2] = \begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix}$$

(d) $\mathbf{A} = \begin{bmatrix} 6 & 3 & -3 \\ -2 & -1 & 2 \\ 16 & 8 & -7 \end{bmatrix}$

- The characteristic polynomial of $\mathbf{A}$ is

$$P_{\mathbf{A}}(\lambda) = \begin{vmatrix} 6-\lambda & 3 & -3 \\ -2 & -1-\lambda & 2 \\ 16 & 8 & -7-\lambda \end{vmatrix} = 3\lambda - 2\lambda^2 - \lambda^3 = -\lambda\,(\lambda+3)\,(\lambda-1)$$

So $\mathbf{A}$ has three distinct real eigenvalues: $\lambda_1 = 0$, $\lambda_2 = -3$ and $\lambda_3 = 1$.

The eigenspace corresponding to the first eigenvector $\lambda_1 = 0$ is the null space of

$$\mathbf{A} - (0)\mathbf{I} = \begin{bmatrix} 6 & 3 & -3 \\ -2 & -1 & 2 \\ 16 & 8 & -7 \end{bmatrix} \quad \rightsquigarrow \quad \begin{bmatrix} 2 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

is the solution set of

$$\begin{array}{l} 2x_1 + x_2 - x_3 = 0 \\ x_3 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad \begin{array}{l} x_1 = 0 \\ x_2 \text{ is unfixed} \\ x_3 = 0 \end{array}$$

So the corresponding eigenvectors are

$$\mathbf{v}_1 \in span\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right)$$

The eigenspace corresponding to the eigenvector $\lambda_2 = -3$ is the null space of

$$\mathbf{A} - (2)\mathbf{I} = \begin{bmatrix} 9 & 3 & -3 \\ -2 & 2 & 2 \\ 16 & 8 & -4 \end{bmatrix} \quad \rightsquigarrow \quad \begin{bmatrix} 3 & 1 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

is the solution set of

$$\begin{array}{lcl} 3x_1 + x_2 - x_3 = 0 & & x_1 = \frac{1}{2}x_3 \\ 2x_2 + x_3 = 0 & \Rightarrow & x_2 = -\frac{1}{2}x_3 \\ 0 = 0 & & x_3 \text{ is unfixed} \end{array}$$

So the corresponding eigenvectors are

$$\mathbf{v}_2 \in span\left(\begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix}\right)$$

The eigenspace corresponding to the first eigenvector $\lambda_3 = 1$ is the null space of

$$\mathbf{A} - (1)\mathbf{I} = \begin{bmatrix} 5 & 3 & -3 \\ -2 & -2 & 2 \\ 16 & 8 & -8 \end{bmatrix} \quad \leftrightsquigarrow \quad \begin{bmatrix} 5 & 3 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

is the solution set of

$$\begin{array}{lcl} 5x_1 + 3x_2 - 3x_3 = 0 & & x_1 = 0 \\ x_2 - x_3 = 0 & \Rightarrow & x_2 = x_3 \\ 0 = 0 & & x_3 \text{ is unfixed} \end{array}$$

So the corresponding eigenvectors are

$$\mathbf{v}_3 \in span\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right)$$

From the eigenvalues of $\mathbf{A}$ we can now form the diagonal matrix $\mathbf{D}$:

$$\mathbf{D} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And from the coresponding eigenvectors we can form the invertible matrix $\mathbf{C}$

$$\mathbf{C} = [\mathbf{v}_1 \mid \mathbf{v}_2 \mid \mathbf{v}_3] = \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ 1 & -\frac{1}{2} & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

such that $\mathbf{D} = \mathbf{C}^{-1}\mathbf{A}\mathbf{C}$.

(e) $\mathbf{A} = \begin{bmatrix} -3 & 10 & -6 \\ 0 & 7 & -6 \\ 0 & 0 & 1 \end{bmatrix}$

- The characteristic polynomial of $\mathbf{A}$ is

$$P_{\mathbf{A}}(\lambda) = \begin{vmatrix} -3-\lambda & 10 & -6 \\ 0 & 7-\lambda & -6 \\ 0 & 0 & 1-\lambda \end{vmatrix} = -(\lambda+3)(\lambda-7)(\lambda-1)$$

So $\mathbf{A}$ has three distinct real eigenvalues: $\lambda_1 = -3$, $\lambda_2 = 7$ and $\lambda_3 = 1$.

The eigenspace corresponding to the first eigenvector $\lambda_1 = 0$ is the null space of

$$\mathbf{A} - (-3)\mathbf{I} = \begin{bmatrix} 0 & 10 & -6 \\ 0 & 10 & -6 \\ 0 & 0 & 4 \end{bmatrix} \quad \leftrightsquigarrow \quad \begin{bmatrix} 0 & 5 & -3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

is the solution set of

$$\begin{array}{lcl} 5x_2 - 3x_3 = 0 & & x_1 \text{ is unfixed} \\ x_3 = 0 & \Rightarrow & x_2 = 0 \\ 0 = 0 & & x_3 = 0 \end{array}$$

So the corresponding eigenvectors are

$$\mathbf{v}_1 \in span\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$$

The eigenspace corresponding to the eigenvector $\lambda_2 = 7$ is the null space of

$$\mathbf{A} - (7)\mathbf{I} = \begin{bmatrix} -10 & 10 & -6 \\ 0 & 0 & -6 \\ 0 & 0 & -6 \end{bmatrix} \quad \leftrightsquigarrow \quad \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

is the solution set of

$$\begin{array}{lcl} x_1 - x_2 = 0 & & x_1 = x_2 \\ x_3 = 0 & \Rightarrow & x_2 \text{ is unfixed} \\ 0 = 0 & & x_3 = 0 \end{array}$$

So the corresponding eigenvectors are

$$\mathbf{v}_2 \in span\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right)$$

The eigenspace corresponding to the first eigenvector $\lambda_3 = 1$ is the null space of

$$\mathbf{A} - (1)\mathbf{I} = \begin{bmatrix} -4 & 10 & -6 \\ 0 & 6 & -6 \\ 0 & 0 & 0 \end{bmatrix} \quad \leftrightsquigarrow \quad \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

is the solution set of

$$\begin{array}{lcl} x_1 - x_3 = 0 & & x_1 = x_3 \\ x_2 - x_3 = 0 & \Rightarrow & x_2 = x_3 \\ 0 = 0 & & x_3 \text{ is unfixed} \end{array}$$

So the corresponding eigenvectors are

$$\mathbf{v}_3 \in span\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$$

From the eigenvalues of $\mathbf{A}$ we can now form the diagonal matrix $\mathbf{D}$:

$$\mathbf{D} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And from the coresponding eigenvectors we can form the invertible matrix $\mathbf{C}$

$$\mathbf{C} = [\mathbf{v}_1 \mid \mathbf{v}_2 \mid \mathbf{v}_3] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

such that $\mathbf{D} = \mathbf{C}^{-1}\mathbf{A}\mathbf{C}$.

2. (Problems 5.2.9 and 5.2.10 in text) Determine whether or not the following matrices are diagonalizable.

(a) $\mathbf{A} = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 0 & -4 \\ 6 & -4 & 3 \end{bmatrix}$

- Yes, because the matrix is real and symmetric. (See Theorem 5.5 in the text.)

(b) $\mathbf{A} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

- Let us calculate the characteristic polynomial of $\mathbf{A}$:

$$P_{\mathbf{A}}(\lambda) = \det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} 3-\lambda & 1 & 0 \\ 0 & 3-\lambda & 1 \\ 0 & 0 & 3-\lambda \end{vmatrix} = (3-\lambda)^3$$

We thus have only one eigenvalue, $\lambda = 3$. The corresponding eigenspace is the null space of

$$\mathbf{A} - (3)\mathbf{I} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

or, equivalently, the solution space of

$$\begin{array}{l} x_2 = 0 \\ x_3 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} x_1 \\ 0 \\ 0 \end{bmatrix} \in span\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$$

So the eigenspace is just 1-dimensional. But we need three linearly independent eigenvectors to construct the matrix $\mathbf{C}$ that diagonalizes $\mathbf{A}$. Hence, $\mathbf{A}$ is not diagonalizable.

3. (Problem 5.2.13 in text) Mark each of the following *True* or *False*.

(a) Every $n \times n$ matrix is diagonalizable.

- False. (An $n \times n$ matrix $\mathbf{A}$ needs n linearly independent eigenvectors in order to be diagonalizable.)

(b) If an $n \times n$ matrix has n distinct real eigenvalues, then it is diagonalizable.

- True. (See Theorem 5.3 in text.)

(c) Every $n \times n$ real symmetric matrix is real diagonalizable.

- True. (See Theorem 5.5 in text.)

(d) An $n \times n$ matrix is diagonalizable if and only if it has n real eigenvalues.

- False. (If it has n distinct real eigenvalues then it is diagonalizable, however it is not absolutely necessary that all the eigenvalues are distinct.)

(e) An $n \times n$ matrix is diagonalizable if and only if the algebraic multiplicity of each of its eigenvalues equals the geometric multiplicity.

- True. (See Theorem 5.4 in text.)

(f) Every invertible matrix is diagonalizable.

- False. (Consider

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This matrix is invertible since $\det(\mathbb{A}) = 1 \neq 0$. However, $\det(\mathbf{A} - \lambda\mathbf{I}) = (1-\lambda)^3$ so there is only one eigenvalue. The correponding eigenspace is the solution space of $(\mathbf{A} - \mathbf{I})\,\mathbf{x} = \mathbf{0}$ which is generated by two vectors $[1, 0, 0]$ and $[0, 1, 0]$. However, we need three independent eigenvectors in order to diagonalize $\mathbf{A}$. Hence, $\mathbf{A}$ is invertible but not diagonalizable.)

(g) Every triagular matrix is diagonalizable.

- False. (See answer to Part (f).)

(h) If $\mathbf{A}$ and $\mathbf{B}$ are similar matrices and $\mathbf{A}$ is diagonalizable, then $\mathbf{B}$ is also diagonalizable.

- True.

(i) If an $n \times n$ matrix $\mathbf{A}$ is diagonalizable, there is a unique diagonal matrix $\mathbf{D}$ that is similar to $\mathbf{A}$.

- False. (Suppose $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of $\mathbf{A}$ and $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are the corresponding set of linearly independent eigenvectors. Then arranging the λ_i along the diagonal of an $n \times n$ matrix we obtain a dialgonalization $\mathbf{D}$ of $\mathbf{A}$. However, if we simply changing the ordering of the eigenvalues, then the same procedure produces a different diagonalization of $\mathbf{A}$.)

(j) If $\mathbf{A}$ and $\mathbf{B}$ are similar square matrices then $\det(\mathbf{A}) = \det(\mathbf{B})$.

- True. (If $\mathbf{A}$ and $\mathbf{B}$ are similar, then, by definition, there is an invertible matrix $\mathbf{C}$ such that $\mathbf{B} = \mathbf{C}^{-1}\mathbf{A}\mathbf{C}$. But then $\det(\mathbf{B}) = \det\left(\mathbf{C}^{-1}\mathbf{A}\mathbf{C}\right) = \det\left(\mathbf{C}^{-1}\right)\det(\mathbf{A})\det(\mathbf{C}) = \det(\mathbf{A})$; since $\det\left(\mathbf{C}^{-1}\right) = 1/\det(\mathbf{C})$).