Math 3013
Homework Set 8

Problems from §5.1 (pgs. 300-301 of text): 2,4,6,8,10,12,17,19,23

1. (Problems 5.1.2, 5.1.4, 5.1.6, 5.1.8, 5.1.10, and 5.1.12 in text). Find the characteristic polynomial, the real eigenvalues, and the corresponding eigenvectors for the following matrices.

(a) $\mathbf{A} = \begin{bmatrix} 7 & 5 \\ -10 & -8 \end{bmatrix}$

- The characteristic polynomial is

$$P(\lambda) = \det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} 7-\lambda & 5 \\ -10 & -8-\lambda \end{vmatrix} = (7-\lambda)(-8-\lambda) - (5)(-10) = \lambda^2 + \lambda - 6 = (\lambda+3)(\lambda-2)$$

The eigenvalues of $\mathbf{A}$ correspond to the roots of $P(\lambda) = 0$; so we have two eigenvalues $\lambda_1 = -3$ and $\lambda_2 = 2$.

The eigenspace corresponding to the eigenvalue $\lambda_1 = -3$ is the solution set of $(\mathbf{A} - (-3)\mathbf{I})\mathbf{x} = 0$; i.e., the null space of the matrix

$$\mathbf{A} - (-3)\mathbf{I} = \begin{bmatrix} 7+3 & 5 \\ -10 & -8+3 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ -10 & -5 \end{bmatrix} \xrightarrow[R_2 \to R_2 - R_1]{R_1 \to \frac{1}{5}R_1} \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$$

The null space of the last matrix is the solution set of

$$\begin{matrix} 2x_1 + x_2 = 0 \\ 0 = 0 \end{matrix} \quad \Rightarrow \quad x_1 = -\frac{1}{2}x_2 \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} -\frac{1}{2}x_2 \\ x_2 \end{bmatrix} \in span\left(\begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}\right)$$

Thus, the eigenvectors corresponding to the eigenvalue $\lambda_1 = -3$ are thus of the form

$$\mathbf{v}_1 = r\begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix} \quad , \quad r \in \mathbb{R} - \{0\}$$

The eigenspace corresponding to the eigenvalue $\lambda_2 = 2$ is the solution set of $(\mathbf{A} - (2)\mathbf{I})\mathbf{x} = 0$; i.e., the null space of the matrix

$$\mathbf{A} - (-3)\mathbf{I} = \begin{bmatrix} 7-2 & 5 \\ -10 & -8-2 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ -10 & -10 \end{bmatrix} \xrightarrow[R_2 \to R_2 + 2R_1]{R_1 \to \frac{1}{5}R_1} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

The null space of the last matrix is the solution set of

$$\begin{matrix} x_1 + x_2 = 0 \\ 0 = 0 \end{matrix} \quad \Rightarrow \quad x_1 = -x_2 \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} -x_2 \\ x_2 \end{bmatrix} \in span\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right)$$

Thus, the eigenvectors corresponding to the eigenvalue $\lambda_2 = 2$ are thus of the form

$$\mathbf{v}_2 = r\begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad , \quad r \in \mathbb{R} - \{0\}$$

(b) $\mathbf{A} = \begin{bmatrix} -7 & -5 \\ 16 & 17 \end{bmatrix}$

- The characteristic polynomial is

$$P(\lambda) = \det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} -7-\lambda & -5 \\ 16 & 17-\lambda \end{vmatrix} = \lambda^2 - 10\lambda - 39 = (\lambda - 13)(\lambda + 3)$$

The eigenvalues of $\mathbf{A}$ correspond to the roots of $P(\lambda) = 0$; so we have two eigenvalues $\lambda_1 = 13$ and $\lambda_2 = -3$.

The eigenspace corresponding to the eigenvalue $\lambda_1 = 13$ is the solution set of $(\mathbf{A} - (13)\mathbf{I})\,\mathbf{x} = 0$; i.e., the null space of the matrix

$$\mathbf{A} - (13)\mathbf{I} = \begin{bmatrix} -20 & -5 \\ 16 & 4 \end{bmatrix} \xrightarrow[R_2 \to R_2 + \frac{4}{5}R_1]{R_1 \to -\frac{1}{5}R_1} \begin{bmatrix} 4 & 1 \\ 0 & 0 \end{bmatrix}$$

The null space of the last matrix is the solution set of

$$\begin{array}{c} 4x_1 + x_2 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad x_1 = -\frac{1}{4}x_2 \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} -\frac{1}{4}x_2 \\ x_2 \end{bmatrix} \in span\left(\begin{bmatrix} -\frac{1}{4} \\ 1 \end{bmatrix}\right)$$

Thus, the eigenvectors corresponding to the eigenvalue $\lambda_1 = 13$ are thus of the form

$$\mathbf{v}_1 = r\begin{bmatrix} -\frac{1}{4} \\ 1 \end{bmatrix} \quad , \quad r \in \mathbb{R} - \{0\}$$

The eigenspace corresponding to the eigenvalue $\lambda_2 = -3$ is the solution set of $(\mathbf{A} - (-3)\mathbf{I})\,\mathbf{x} = 0$; i.e., the null space of the matrix

$$\mathbf{A} - (-3)\mathbf{I} = \begin{bmatrix} -4 & -5 \\ 16 & 20 \end{bmatrix} \xrightarrow[R_2 \to R_2 + 4R_1]{R_1 \to -R_1} \begin{bmatrix} 4 & 5 \\ 0 & 0 \end{bmatrix}$$

The null space of the last matrix is the solution set of

$$\begin{array}{c} 4x_1 + 5x_2 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad x_1 = -\frac{5}{4}x_2 \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} -\frac{5}{4}x_2 \\ x_2 \end{bmatrix} \in span\left(\begin{bmatrix} -\frac{5}{4} \\ 1 \end{bmatrix}\right)$$

Thus, the eigenvectors corresponding to the eigenvalue $\lambda_2 = -3$ are thus of the form

$$\mathbf{v}_2 = r\begin{bmatrix} -\frac{5}{4} \\ 1 \end{bmatrix} \quad , \quad r \in \mathbb{R} - \{0\}$$

(c) $\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix}$

- The characteristic polynomial is

$$P(\lambda) = \det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} 1-\lambda & -2 \\ 1 & 2-\lambda \end{vmatrix} = \lambda^2 - 3\lambda + 4$$

The roots of this (quadratic) polynomial are given by the quadratic formula:

$$\lambda = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(4)}}{2(1)} = \frac{3 \pm \sqrt{9-16}}{2} = \frac{3}{2} \pm \frac{\sqrt{-7}}{2} = \frac{3}{2} \pm \sqrt{7}i$$

Thus, we have two complex roots. Lacking a real eigenvalue, the problem ends here.

(d) $\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ -4 & 2 & -1 \\ 4 & 0 & 3 \end{bmatrix}$

- The characteristic polynomial is

$$\begin{aligned} P(\lambda) &= \det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} -1-\lambda & 0 & 0 \\ -4 & 2-\lambda & -1 \\ 4 & 0 & 3-\lambda \end{vmatrix} \\ &= (-1-\lambda)\begin{vmatrix} 2-\lambda & -1 \\ 0 & 3-\lambda \end{vmatrix} - (0)\begin{vmatrix} -4 & -1 \\ 4 & 3-\lambda \end{vmatrix} + (0)\begin{vmatrix} -4 & 2-\lambda \\ 4 & 0 \end{vmatrix} \\ &= (-1-\lambda)\left((2-\lambda)(3-\lambda) - 0\right) - 0 + 0 \\ &= -(\lambda+1)(\lambda-2)(\lambda-3) \end{aligned}$$

We thus have three eigenvalues: $\lambda_1 = -1$, $\lambda_2 = 2$, and $\lambda_3 = 3$.

The eigenspace corresponding to the eigenvalue $\lambda_1 = -1$ correspond to the null space of

$$\mathbf{A} - (-1)\mathbf{I} = \begin{bmatrix} 0 & 0 & 0 \\ -4 & 3 & -1 \\ 4 & 0 & 4 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

i.e. the solution set of

$$\begin{array}{c} x_1 + x_3 = 0 \\ x_2 + x_3 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad \begin{array}{c} x_1 = -x_3 \\ x_2 = -x_3 \end{array} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \end{bmatrix} \in span\left(\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}\right)$$

Thus, the eigenvectors corresponding to the eigenvalue $\lambda_1 = -1$ will be vectors of the form

$$\mathbf{v}_1 = r\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \quad , \quad r \in \mathbb{R} - \{0\}$$

The eigenspace corresponding to the eigenvalue $\lambda_2 = 2$ correspond to the null space of

$$\mathbf{A} - (2)\mathbf{I} = \begin{bmatrix} -3 & 0 & 0 \\ -4 & 0 & -1 \\ 4 & 0 & 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

i.e. the solution set of

$$\begin{array}{c} x_1 = 0 \\ x_3 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} 0 \\ -x_2 \\ 0 \end{bmatrix} \in span\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right)$$

Thus, the eigenvectors corresponding to the eigenvalue $\lambda_2 = 2$ will be vectors of the form

$$\mathbf{v}_2 = r\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad , \quad r \in \mathbb{R} - \{0\}$$

The eigenspace corresponding to the eigenvalue $\lambda_3 = 3$ correspond to the null space of

$$\mathbf{A} - (3)\mathbf{I} = \begin{bmatrix} -4 & 0 & 0 \\ -4 & -1 & -1 \\ 4 & 0 & 0 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

i.e. the solution set of

$$\begin{array}{c} x_1 = 0 \\ x_2 + x_3 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad \begin{array}{c} x_1 = 0 \\ x_2 = -x_3 \end{array} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} 0 \\ -x_3 \\ x_3 \end{bmatrix} \in span\left(\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}\right)$$

Thus, the eigenvectors corresponding to the eigenvalue $\lambda_3 = 3$ will be vectors of the form

$$\mathbf{v}_3 = r\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \quad , \quad r \in \mathbb{R} - \{0\}$$

(e) $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ -8 & 4 & -5 \\ 8 & 0 & 9 \end{bmatrix}$

- The characteristic polynomial is

$$P(\lambda) = \det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} 1-\lambda & 0 & 0 \\ -8 & 4-\lambda & -5 \\ 8 & 0 & 9-\lambda \end{vmatrix} = (1-\lambda)(4-\lambda)(9-\lambda)$$

So we have three possible eigenvalues : $\lambda_1 = 1$, $\lambda_2 = 4$, and $\lambda_3 = 9$. The corresponding eigenvectors are calculated as in the preceding problems:

$$\begin{array}{lcl} \lambda_1 = 1 & \Rightarrow & \mathbf{v}_1 = r\,[-1,-1,1] \quad , \quad r \in \mathbb{R} - \{0\} \\ \lambda_2 = 4 & \Rightarrow & \mathbf{v}_2 = r\,[0,1,0] \quad , \quad r \in \mathbb{R} - \{0\} \\ \lambda_3 = 9 & \Rightarrow & \mathbf{v}_3 = r\,[0,-1,1] \quad , \quad r \in \mathbb{R} - \{0\} \end{array}$$

(f) $\mathbf{A} = \begin{bmatrix} -4 & 0 & 0 \\ -7 & 2 & -1 \\ 7 & 0 & 3 \end{bmatrix}$

- The characteristic polynomial is

$$P(\lambda) = \det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} -4-\lambda & 0 & 0 \\ -7 & 2-\lambda & -1 \\ 7 & 0 & 3-\lambda \end{vmatrix} = (2-\lambda)(3-\lambda)(-4-\lambda)$$

So we have three real eigenvalues: $\lambda_1 = 2$, $\lambda_2 = 3$, and $\lambda_3 = -4$. The corresponding eigenvectors (calculated as in the preceding problems) are

$$\begin{array}{lcl} \lambda_1 = 2 & \Rightarrow & \mathbf{v}_1 = r\,[0,1,0] \quad , \quad r \in \mathbb{R} - \{0\} \\ \lambda_2 = 3 & \Rightarrow & \mathbf{v}_2 = r\,[0,1,-1] \quad , \quad r \in \mathbb{R} - \{0\} \\ \lambda_3 = -4 & \Rightarrow & \mathbf{v}_3 = r\,[-1,-1,1] \quad , \quad r \in \mathbb{R} - \{0\} \end{array}$$

2. (Problems 5.1.17 and 5.1.19 in text) Find the eigenvalues λ_i and the corresponding eigenvectors $\mathbf{v}_i$ for the following linear transformations.

(a) $T([x,y]) = [2x-3y, -3x+2y]$

- First we calculate the matrix corresponding to T:

$$\begin{array}{l} T([1,0]) = [2,-3] \\ T([0,1]) = [-3,2] \end{array} \quad \Rightarrow \quad \mathbf{A}_T = \begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix}$$

The characteristic polynomial for this matrix is

$$P(\lambda) = \det(\mathbf{A}_T - \lambda\mathbf{I}) = \begin{vmatrix} 2-\lambda & -3 \\ -3 & 2-\lambda \end{vmatrix} = \lambda^2 - 4\lambda - 5 = (\lambda-5)(\lambda+1)$$

We thus have two real eigenvalues: $\lambda_1 = 5$ and $\lambda_2 = -1$.

The eigenspace corresponding to the eigenvalue $\lambda_1 = 5$ is the null space of

$$\begin{vmatrix} 2-5 & -3 \\ -3 & 2-5 \end{vmatrix} = \begin{vmatrix} -3 & -3 \\ -3 & -3 \end{vmatrix} \rightsquigarrow \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}$$

which is the solution set of

$$\begin{array}{c} x_1 + x_2 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} -x_2 \\ x_2 \end{bmatrix} \in span\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right)$$

Hence, the corresponding eigenvectors are of the form

$$\mathbf{v}_1 = r\begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad , \quad r \in \mathbb{R} - \{0\}$$

The eigenspace corresponding to the eigenvalue $\lambda_1 = -1$ is the null space of

$$\begin{vmatrix} 2-(-1) & -3 \\ -3 & 2-(-1) \end{vmatrix} = \begin{vmatrix} 3 & -3 \\ -3 & 3 \end{vmatrix} \rightsquigarrow \begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix}$$

which is the solution set of

$$\begin{array}{c} x_1 - x_2 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} x_2 \\ x_2 \end{bmatrix} \in span\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$$

Hence, the corresponding eigenvectors are of the form

$$\mathbf{v}_2 = r\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad , \quad r \in \mathbb{R} - \{0\}$$

(b) $T([x_1, x_2, x_3]) = [x_1 + x_3, x_2, x_1 + x_3]$

- First we calculate the matrix corresponding to this linear transfomation:

$$\begin{array}{r} T([1,0,0]) = [1,0,1] \\ T([0,1,0]) = [0,1,0] \\ T([0,0,1]) = [1,0,1] \end{array} \quad \Rightarrow \quad \mathbf{A}_T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

The characteristic polynomial for this matrix is

$$P(\lambda) = \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{vmatrix} = -\lambda(-1+\lambda)(\lambda-2)$$

We thus have three possible eigenvalues: $\lambda_1 = 0$, $\lambda_2 = 1$, and $\lambda_2 = 2$.

The eigenspace corresponding to $\lambda_1 = 0$ will be the null space of

$$\begin{bmatrix} 1-0 & 0 & 1 \\ 0 & 1-0 & 0 \\ 1 & 0 & 1-0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad \rightsquigarrow \quad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

or, equivalently, the solution set of

$$\begin{array}{r} x_1 + x_3 = 0 \\ x_2 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} -x_3 \\ 0 \\ x_3 \end{bmatrix}$$

so the correpsponding eigenvectors will be of the form

$$\mathbf{v}_1 = r\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad , \quad r \in \mathbb{R} - \{0\}$$

The eigenspace corresponding to $\lambda_1 = 1$ will be the null space of

$$\begin{bmatrix} 1-1 & 0 & 1 \\ 0 & 1-1 & 0 \\ 1 & 0 & 1-1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \rightsquigarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

or, equivalently, the solution set of

$$\begin{array}{r} x_1 = 0 \\ x_3 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} 0 \\ x_2 \\ 0 \end{bmatrix}$$

so the correpsponding eigenvectors will be of the form

$$\mathbf{v}_2 = r\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad , \quad r \in \mathbb{R} - \{0\}$$

The eigenspace corresponding to $\lambda_1 = 2$ will be the null space of

$$\begin{bmatrix} 1-2 & 0 & 1 \\ 0 & 1-2 & 0 \\ 1 & 0 & 1-2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad \rightsquigarrow \quad \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

or, equivalently, the solution set of

$$\begin{array}{r} x_1 - x_3 = 0 \\ x_2 = 0 \\ 0 = 0 \end{array} \quad \Rightarrow \quad \mathbf{x} = \begin{bmatrix} x_3 \\ 0 \\ x_3 \end{bmatrix}$$

so the correpsponding eigenvectors will be of the form

$$\mathbf{v}_3 = r \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad , \quad r \in \mathbb{R} - \{0\}$$

3. Mark each of the following statements *True* or *False*.

(a) Every square matrix has real eigenvalues.

- False. (Counterexample: the characteristic polynomial for the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is $\lambda^2 + 1$, whose only roots are $\pm i$.)

(b) Every $n \times n$ matrix has n distinct (possible complex) eigenvalues.

- False. (Counterexample: the eigenvalues of an identity matrix are all 1's).

(c) Every $n \times n$ matrix has n not necessarily distinct and possibly complex eigenvalues.

- True.

(d) There can be only one eigenvalue associated with an eigenvector of a linear transformation.

- True.

(e) There can be only one eigenvector associated with an eigenvalue of a linear transformation.

- False. (See Part (b).)

(f) If $\mathbf{v}$ is an eigenvector of a matrix $\mathbf{A}$, then $\mathbf{v}$ is an eigenvector of $\mathbf{A} + c\mathbf{I}$ for all scalars c.

- True. (If $\mathbf{v}$ is an eigenvalue of $\mathbf{A}$, then $(\mathbf{A} + c\mathbf{I})\,\mathbf{v} = \mathbf{A}\mathbf{v} + c\mathbf{I}\mathbf{v} = \lambda\mathbf{v} + c\mathbf{v} = (\lambda + c)\,\mathbf{v}$. So $\mathbf{v}$ is an eigenvector of $\mathbf{A} + c\mathbf{I}$ with eigenvalue $\lambda + c$.)

(g) If λ is an eigenvalue of a matrix $\mathbf{A}$, then λ is an eigenvalue of $\mathbf{A} + c\mathbf{I}$ for all scalars c.

- False. (See Part (f).)

(h) If $\mathbf{v}$ is an eigenvector of an invertible matrix $\mathbf{A}$, then $c\mathbf{v}$ is an eigenvector of $\mathbf{A}^{-1}$ for all non-zero scalars c.

- True. (Note $c\mathbf{v} = \mathbf{I}\,(c\mathbf{v}) = c\mathbf{I}\mathbf{v} = c\left(\mathbf{A}^{-1}\mathbf{A}\right)\mathbf{v} = c\mathbf{A}^{-1}\,(\mathbf{A}\mathbf{v}) = c\mathbf{A}^{-1}\,(\lambda\mathbf{v}) = \lambda\mathbf{A}^{-1}\,(c\mathbf{v})$. Hence, if $\lambda \neq 0$,

$$\mathbf{A}^{-1}\,(c\mathbf{v}) = \frac{1}{\lambda}\,(c\mathbf{v})$$

  In fact, we're guaranteed that $\lambda \neq 0$ by the fact that $\mathbf{A}$ is invertible. So $c\mathbf{v}$ will be an eigenvector of $\mathbf{A}^{-1}$ with eigenvalue λ^{-1}.)

(i) Every vector in a vector space V is an eigenvector of the identity transformation of V into V.

- False. (To meet the definition of a eigenvectors a vector must be non-zero.)

(j) Ever nonzero vector in a vector space V is an eigenvector of the identity transformation of V into V.

- True.