Math 2233 Sample Final
Summer 2017

1. Solve the following initial value problem

$$y' + \frac{3}{x}y = x \qquad , \qquad y(1) = 1$$

2. Use the Method of Variation of Parameters to find the general solution of

$$y'' - 4y = 3e^{x} \quad .$$

3. Suppose $y(x) = \sum_{n=0}^{\infty} a_n (x-1)^n$. Find a power series expression for $x^2 y''$.

4. Determine the recursion relations for the power series solution $y(x) = \sum_{n=0}^{\infty} a_n x^n$ (about $x = 0$) for the following differential equation

$$y'' + xy' + 2y = 0$$

5. Consider the following initial value problem

$$xy'' - y = 0 \ , \ y(1) = 1 \ , \ y'(1) = 2$$

Given that the recursion relations for a power series solution of the form $\displaystyle\sum_{n=0}^{\infty} a_n (x-1)^n$ are

$$a_{n+2} = \frac{a_n - n(n+1)a_{n+1}}{(n+2)(n+1)}$$

write down (explicitly) the first four terms (i.e. up to order $(x-1)^3$) of this power series solution.

6. Consider the differential equation $x(x-2)^2 y'' + (x+2)y' + (x-1)y = 0$.

(a) Identify and classify the singular points of this differential equation.

(b) What is the minimal radius of convergence of a power series solution of this equation about the point $x = 4$.

7. Find a function having the following Laplace transform:

$$\frac{7}{s^2+9} + \frac{1}{s^5}$$

8. Solve the following differential equation using Laplace transforms:

$$y'' + 2y' - 3y = 0 \quad ; \quad y(0) = 1 \quad , \quad y'(0) = 0 \quad .$$