

Math 2233
Homework Set 7

1. Find the first four terms of the Taylor expansion about $x = 0$ of the solution of

$$\begin{aligned}y' &= y^2 \\ y(0) &= 1\end{aligned}$$

2. Find the first four terms of the Taylor expansion about $x = 1$ of the solution of

$$\begin{aligned}y' &= x^2y \\ y(1) &= 1\end{aligned}$$

3. Transform each of the following expressions into a single power series.

$$\begin{aligned}\text{(a)} \quad & \sum_{n=1}^{\infty} (n+1)(x-1)^{n-1} + \sum_{n=0}^{\infty} n(x-1)^n \\ \text{(b)} \quad & \sum_{n=0}^{\infty} (n+1)a_{n+2}x^{n+1} + \sum_{n=0}^{\infty} na_nx^{n-1} \\ \text{(c)} \quad & (x-1) \sum_{n=0}^{\infty} na_nx^{n-1} + \sum_{n=0}^{\infty} a_nx^n \\ \text{(d)} \quad & x \sum_{n=0}^{\infty} na_n(x-1)^{n-1} + \sum_{n=0}^{\infty} a_n(x-1)^n \\ \text{(e)} \quad & x^2 \sum_{n=0}^{\infty} n(n-1)a_n(x-1)^{n-2}\end{aligned}$$

4. Find the recursion relations for the power series solutions $y(x) = \sum_{n=0}^{\infty} a_n(x-x_o)^n$ of the following differential equations

$$\begin{aligned}\text{(a)} \quad & y'' - xy' - y = 0, \quad x_o = 0 \\ \text{(b)} \quad & y'' - xy' - y = 0, \quad x_o = 1 \\ \text{(c)} \quad & (1-x)y'' + y = 0, \quad x_o = 0 \\ \text{(d)} \quad & y'' + xy' + 2y = 0, \quad x_o = 0 \\ \text{(e)} \quad & (1+x^2)y'' - 4xy' + 6y = 0, \quad x_o = 0\end{aligned}$$

5. Find power series expressions for the general solutions of the following differential equations. (You may utilize recursion relations found in Problem 2.)

$$\begin{aligned}\text{(a)} \quad & y'' - xy' - y = 0, \quad x_o = 0 \\ \text{(b)} \quad & y'' - xy' - y = 0, \quad x_o = 1\end{aligned}$$

6. Find power series expressions for the solutions to the following initial value problems. (You may utilize recursion relations found in Problem 2.)

$$\begin{aligned}\text{(a)} \quad & (1-x)y'' + y = 0, \quad y(0) = 2, \quad y'(0) = 1 \\ \text{(b)} \quad & y'' - xy' - y = 0, \quad y(1) = 1, \quad y'(1) = 2\end{aligned}$$