Math 2233
Homework Set 6

1. Solve the following Euler-type equations.

(a) $x^2y'' + xy' + y = 0$

(b) $x^2y'' - xy' + 2y = 0$

(c) $4x^2y'' - 4xy' + 3y = 0$, $y(1) = 0$, $y'(1) = 1$

(d) $x^2y'' - 3xy' + 3y = 0$

(e) $x^2y'' + 5xy' + 5y = 0$

2. Given that $y_1(x) = e^x$ and $y_2(x) = x$ are solutions of

$$(1 - x)y'' + xy' - y = 0$$

find the general solution to

$$(1 - x)y'' + xy' - y = 2(x - 1)^2e^{-x}$$

by the method of Variation of Parameters.

3. Find the general solution of each of the following equations by the method of Variation of Parameters. If initial conditions are given, find the solution satisfying that initial value problem.

(a) $y'' - 3y' + 2y = 10$

(b) $y'' + y = \sin(x)$, $y(0) = 1$, $y'(0) = 2$

(c) $y'' - 7y' + 10y = 100x$

(d) $y'' + 4y = \sec(2x)$

4. Use the method of Variation of Parameters to solve the following non-homogeneous Euler-type equation.

$$x^2y'' - 5xy' + 9y = x^3$$

5. Find the general solution to the following differential equations. If initial conditions are specified, also determine the solution satisfying those initial conditions.

(a) $y^{(4)} + 2y'' + y = 0$

(b) $y''' - y'' - y' + y = 0$

(c) $y''' - 3y'' + 3y' - y = 0$, $y(0) = 1$, $y'(0) = 2$, $y''(0) = 3$

(d) $y''' + 5y'' - y' - 5y = 0$

(e) $y^{(4)} - 9y'' = 0$