Math 2233
Homework Set 5

1 Determine whether the given equation is linear or nonlinear. If it is linear, write it in standard form and state whether it is homogeneous or non-homogeneous.

(a) $xy'' + 2x^3y' + y = 0$

(b) $y'' + xy' + y^2 = 2x$

(c) $3y'' + 2y' + y = x^5$

2. Verify that the two given functions are linearly independent solutions of the given homogeneous equation and then find the general solution.

(a) $y'' + 9y = 0$, $y_1(x) = \sin(3x)$, $y_2(x) = \cos(3x)$

(b) $y'' + 2y' - 15y = 0$, $y_1(x) = e^{3x}$, $y_2(x) = e^{-5x}$

(c) $y'' + 4y' + 4y = 0$, $y_1(x) = e^{-2x}$, $y_2(x) = xe^{-2x}$

3. Given that $y_1(x) = e^{3x}$ is one solution of $y'' - 5y' + 6y = 0$, find a second linearly independent solution and then write down the general solution.

4. Given that $y_1(x) = e^{2x}$ is one solution of $y'' - 4y = 0$, find a second linearly independent solution and then write down the general solution.

5. Given that $y_1(x) = x$ is one solution of $x^2y'' - 2xy' + 2y = 0$, find a second linearly independent solution and then write down the general solution.

6. Given that $y_1(x) = x\sin(x)$ is one solution of $x^2y'' - 2xy' + (x^2 + 2)y = 0$, find a second linearly independent solution and then write down the general solution.

7. Find the general solution of the following differential equations

(a) $y'' - 5y = 0$.

(b) $y'' - 3y' + 2y = 0$

(c) $y'' - y' - 20y = 0$

(d) $y'' - 13y' + 42y = 0$

(e) $y'' + y' + 7y = 0$

(f) $y'' + 2y' + 5y = 0$

8. Solve the following initial value problems.

(a) $y'' - 9y = 0$, $y(0) = 1$, $y'(0) = 2$.

(b) $y'' - 2y' + y = 0$, $y(0) = 2$, $y'(0) = 1$.

(c) $y'' + 2y' + 2y = 0$, $y(0) = 1$, $y'(0) = -1$