Math 2233
Homework Set 1

1. Determine the order of the following differential equations, whether or not the equations are linear and whether the differential equations are ODEs (ordinary differential equations) or PDEs (partial differential equations).

(a) $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + 2y = \sin(x)$

(b) $(1+y^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = e^x$

(c) $\dfrac{\partial^2\phi}{\partial x^2} + y^2\dfrac{\partial\phi}{\partial x} = x^2$

(d) $\dfrac{d^4y}{dx^4} + \dfrac{d^3y}{dx^3} + \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + y = 1$

(e) $\dfrac{dy}{dx} + xy^2 = 0$

(f) $\dfrac{\partial^2\phi}{\partial x\partial y} + \dfrac{\partial\phi}{\partial x}\phi = x^2$

(g) $\dfrac{d^2y}{dx^2} + \sin(x+y) = \sin(x)$

2.

(a) Plot the direction field for the differential equation

$$y' = x^2 + y.$$

(b) Sketch the solution that satisfies $y(0) = 2$.

(c) Sketch the solution that satisfies $y(0) = 1$.

3. Consider the differential equation $y' = (y+2)*(y-2)$. What can you say about the behavior of solution $y(x)$ that passes through the point $x = 0$, $y = -1$ as $x \to \infty$? (Hint: the sign of the right hand side of the differential equations tells you whether or not a solution y(x) is increasing or decreasing.)

4. Using the Euler Method, find an approximate value for $y(1)$ for the following initial value problem (take $h = \Delta x = 0.2$):

$$\frac{dy}{dx} = x + y \quad , \quad y(0) = 1$$

5. Using the Euler Method, find an approximate value for $y(1.5)$ for the following initial value problem (take $h = \Delta x = 0.1$):

$$\frac{dy}{dx} = xe^y \quad , \quad y(1) = 0$$