Math 2233

FINAL EXAM

10:00-11:50, Friday, May 6, 2005

Name:________________________________

1. (15 pts) Solve the following initial value problem.

$$xy' - 3y = x^2 \qquad , \qquad y(1) = 3 \quad .$$

2. Consider the following differential equation.

$$y + 6x^2 + \left(x\ln(x) - 4xe^{2y}\right)y' = 0.$$

(a) (5 pts) Verify that $\mu = 1/x$ is an integrating factor for this equation.

(b) (10 pts) Find an implicit solution for this differential equation.

3. (15 pts) Find an explicit solution of the following homogeneous ODE: $\dfrac{dy}{dx} = \dfrac{y^2 + yx}{x^2}$.
(Hint: use the following change of variables: $z = y/x$.)

4. Find the general solutions of the following differential equations.

(a) (5 pts) $y'' - 3y' + 3y = 0$

(b) (5 pts) $x^2y'' - 5xy' + 9y = 0$

(c) (5 pts) $y'''' + 4y''' + 4y'' = 0$.

5. (10 pts) Given that $y_1(x) = x$ is one solution of $x^2y'' - xy' + y = 0$, use Reduction of Order to determine the general solution.

6. (15 pts) Use the Method of Variation of Parameters to find the general solution of the following inhomogeneous differential equation.

$$y'' - 5y' + 6y = e^x \quad .$$

7. (10 pts) Suppose $y = \sum_{n=0}^{\infty} a_n (x-1)^n$ is a series solution of $xy'' + 2y = 0$. Determine the recursion relations for the coefficients $\{a_n\}$.

8. (10 pts) Suppose that $y(x) = \sum_{n=0}^{\infty} a_n x^n$ is a power series solution of $y'' - xy' - y = 0$. Given that the recursion relations for the coefficients $\{a_n\}$ are

$$a_{n+2} = \frac{a_n}{n+2} \quad ,$$

write down the first four terms of the power series solution satisfying $y(0) = 3$, $y'(0) = 2$.

9. Consider the differential equation $x^2(x+2)^2 y'' - 2xy' + 3y = 0$

(a) (10 pts) Identify and classify the singular points of this differential equation.

(b) (5 pts) What is the minimal radius of convergence of a power series solution of the form $y(x) = \sum_{n=0}^{\infty} a_n (x-4)^n$?

10. (15 pts) Consider the differential equation

$$3xy'' + y = 0$$

and suppose there is a series solution of the form

$$y(x) = x^r \sum_{n=0}^{\infty} a_n x^n \quad \text{with } a_0 \neq 0 \quad .$$

Determine the leading exponents (the possible values for r) and the recursion relations for the coefficients a_n. (You **do not** have to write down the corresponding solutions.)

11. (15 pts) Use the Laplace Transform Method to solve

$$\begin{aligned} y'' + 3y' + 2y &= 0 \\ y(0) &= 3 \\ y'(0) &= -4 \end{aligned}$$

Table of Laplace Transforms

$$\mathcal{L}\left[x^n\right] = \frac{n!}{s^{n+1}}$$
$$\mathcal{L}\left[e^{ax}\right] = \frac{1}{s-a}$$
$$\mathcal{L}\left[\sin(ax)\right] = \frac{a}{s^2+a^2}$$
$$\mathcal{L}\left[\cos(ax)\right] = \frac{s}{s^2+a^2}$$
$$\mathcal{L}\left[\sinh\left(ax\right)\right] = \frac{a}{s^2-a^2}$$
$$\mathcal{L}\left[\cosh\left(ax\right)\right] = \frac{s}{s^2-a^2}$$
$$\mathcal{L}\left[e^{ax}\sin\left(bx\right)\right] = \frac{b}{\left(s-a\right)^2+b^2}$$
$$\mathcal{L}\left[e^{ax}\cos\left(bx\right)\right] = \frac{s-a}{\left(s-a\right)^2+b^2}$$
$$\mathcal{L}\left[x^n e^{ax}\right] = \frac{n!}{\left(s-a\right)^{n+1}}$$