Math 2233
FINAL EXAM
8:s0 – 10:20 , May 11, 2001

Name:________________________________

1. (15 pts) Solve the following initial value problem.

$$xy' + 3y = 5x \qquad , \qquad y(1) = 3 \quad .$$

2. (15 pts) Find an implicit solution of the following initial value problem.

$$(y/x + 4x)\,dx + (\ln(x) - 3)\,dy = 0 \qquad , \qquad y(1) = 1$$

(Hint: the equation is exact.)

3.

(a)(5 pts) Show that the following differential equation is not exact.

$$y + (2x - ye^y)\, y' = 0$$

(b) (10 pts) Find an integrating factor for the differential equation in (a). (Hint: look for an integrating factor that depends only on y.)

4. (10 pts) Find an implicit solution of the following ODE.

$$\frac{dy}{dx} = \frac{y^2 + yx}{x^2} \quad .$$

(Hint: try the change of variables $z = y/x$.)

5. (15 pts) Given that $y_1(x) = x^{-1}$ is one solution of $x^2y'' + 3xy' + y = 0$, use Reduction of Order to determine the general solution and the solution satisfying $y(1) = 1$, $y'(1) = 0$.

6. (10 pts) Use the Method of Variation of Parameters to find the general solution of the following inhomogeneous differential equation.

$$y'' - 3y' + 2y = e^{3x} \quad .$$

7. (10 pts) What is the minimal radius of convergence of a power series solution of

$$(1 + x^2)y'' + 2y' + xy = 0$$

about $x_o = 2$?

8.(15 pts) Find the recursion relations for a power series solution about $x_o = 1$ for the following differential equation.

$$xy'' - 2y = 0$$

9. (15 pts) Given that the recursion relations for $y'' - xy' + y = 0$ about $x_o = 0$ are

$$a_{n+2} = \frac{(n-1)a_n}{(n+2)(n+1)} \qquad , \qquad n = 0, 1, 2, 3, \ldots$$

Write down the first 4 terms (i.e., to order x^3) for the power series solution satisfying $y(0) = 1$, $y'(0) = 2$.

10. (15 pts) Consider the following linear differential equation $2xy'' - 2y = 0$. Assume a solution of the form $y = x^r \sum_{n=0}^{in} a_n x^n$ and determine the possible values of r.

11. (15 pts) Use the Laplace Transform technique to solve the following initial value problem.

$$y'' - 2y' - 3y = 0 \qquad , y(0) = 3 \quad , \quad y'(0) = 1$$

Table of Laplace Transforms

$$\mathcal{L}\left[x^n\right] = \frac{n!}{s^{n+1}}$$
$$\mathcal{L}\left[e^{ax}\right] = \frac{1}{s-a}$$
$$\mathcal{L}\left[\sin(ax)\right] = \frac{a}{s^2+a^2}$$
$$\mathcal{L}\left[\cos(ax)\right] = \frac{s}{s^2+a^2}$$
$$\mathcal{L}\left[\sinh\left(ax\right)\right] = \frac{a}{s^2-a^2}$$
$$\mathcal{L}\left[\cosh\left(ax\right)\right] = \frac{s}{s^2-a^2}$$
$$\mathcal{L}\left[e^{ax}\sin\left(bx\right)\right] = \frac{b}{\left(s-a\right)^2+b^2}$$
$$\mathcal{L}\left[e^{ax}\cos\left(bx\right)\right] = \frac{s-a}{\left(s-a\right)^2+b^2}$$
$$\mathcal{L}\left[x^n e^{ax}\right] = \frac{n!}{\left(s-a\right)^{n+1}}$$