

Math 2233
Homework Set 7

1. Find the general solution to the following differential equations. If initial conditions are specified, also determine the solution satisfying those initial conditions.

- (a) $y^{(4)} + 2y'' + y = 0$
- (b) $y''' - y'' - y' + y = 0$
- (c) $y''' - 3y'' + 3y' - y = 0, y(0) = 1, y'(0) = 2, y''(0) = 3$
- (d) $y''' + 5y'' - y' - 5y = 0$
- (e) $y^{(4)} - 9y'' = 0$

2. Combine each of the following power series expressions into a single power series.

- (a) $\sum_{n=0}^{\infty} (n+1)(x-1)^{n-1} + \sum_{n=0}^{\infty} n(x-1)^n$
- (b) $\sum_{n=0}^{\infty} (n+1)a_{n+2}x^{n+1} + \sum_{n=0}^{\infty} na_nx^{n-1}$
- (c) $(x-1) \sum_{n=0}^{\infty} na_nx^{n-1} + \sum_{n=0}^{\infty} a_nx^n$
- (d) $x \sum_{n=0}^{\infty} na_n(x-1)^{n-1} + \sum_{n=0}^{\infty} a_n(x-1)^n$
- (e) $x^2 \sum_{n=0}^{\infty} n(n-1)a_n(x-1)^{n-2}$

3. Find the recursion relations for the power series solutions $y(x) = \sum_{n=0}^{\infty} a_n(x-x_o)^n$ of the following differential equations

- (a) $y'' - xy' - y, x_o = 0$
- (b) $y'' - xy' - y = 0, x_o = 1$
- (c) $(1-x)y'' + y = 0, x_o = 0$
- (d) $y'' + xy' + 2y = 0, x_o = 0$
- (e) $(1+x^2)y'' - 4xy' + 6y = 0, x_o = 0$

4. Find power series expressions for the general solutions of the following differential equations. (You may utilize recursion relations found in Problem 3.)

- (a) $y'' - xy' - y = 0, x_o = 0$
- (b) $y'' - xy' - y = 0, x_o = 1$

5. Find power series expressions for the solutions to the following initial value problems. (You may utilize recursion relations found in Problem 3.)

- (a) $(1-x)y'' + y = 0, y(0) = 2, y'(0) = 1$
- (b) $y'' - xy' - y = 0, y(1) = 1, y'(1) = 2$