

# Syllabus for the Comprehensive Exam in Topology

## Geometric Topology

Preparatory Course: Math 5313

1. Compact, connected 2-manifolds,  $n$ -manifolds, trees, graphs and Euler characteristic.
2. The fundamental group, retractions and deformation retractions, the fundamental group of a product space, homotopy equivalence and simple connectivity.
3. Basic combinatorial group theory: free groups, free products and presentations.
4. Seifert and Van Kampen theorem, computations of fundamental groups of compact, connected surfaces and CW-complexes.
5. Covering spaces, lifting theorems, regular covering spaces and quotient spaces of properly discontinuous group actions, classification of covering spaces.
6. Applications of homotopy theory: the Brouwer fixed point theorem and the Borsuk-Ulam theorem in dimension two, applications to knot theory.

## Algebraic Topology

Preparatory Course: Math 6323

1. Definition of singular homology groups, the exact sequence of a pair, homotopy invariance, excision property, Mayer-Vietoris sequence.
2. Computations of homology groups of finite graphs and manifolds, the Jordan-Brouwer separation theorem, relation between the fundamental group and the first homology group.
3. Simplicial homology, cellular and singular homology of a CW-complex, homological definition of Euler characteristic.
4. Homology with arbitrary coefficients, the universal coefficient theorem.
5. Definition of cohomology groups, the universal coefficient theorem, excision property, the Mayer-Vietoris sequence.

REFERENCES: William S. Massey, *Algebraic Topology: An Introduction*, GTM 56; William S. Massey, *A Basic Course in Algebraic Topology*, GTM 127; Allen Hatcher, *Algebraic Topology*.