Syllabus of PhD Comprehensive Exam: Numerical Analysis

I. Preparatory Courses:

1. Math 5543 Numerical analysis for differential equations.
2. Math 5553 Numerical analysis for linear algebra.
3. Math 4513 Numerical analysis.

II. Subjects:

1. Numerical methods for Ordinary Differential Equations and their stabilty and convergence theory. These topics are covered in Math 4513 which is a prerequisite for Math 5543.
   (a) One-step methods.
   1. Explicit: Taylor method (not to be confused with Taylor series), Runge Kutta.
   2. Implicit: Backward Euler, Trapezoidal.
   (b) Multi-step methods.
   1. Explicit: Adams Bashforth.
   2. Implicit: Adams Moulton, Backward differentiation.
   (c) Introduction to stability theorey.
2. Numerical methods (finite difference method) for parabolic equations in one and two spacial dimensions; for hyperbolic equations in one spacial dimension.

3. Numerical methods (mostly finite difference method, brief understanding of finite element method) for linear second order elliptic equations in one and two space dimensions.

4. Theory of consistence, convergence and stability. Lax equivalence theorem, von Neumann stability analysis. Error analyses using maximum principles and energy method.

5. Iterative methods for solving linear algebraic equations. Basic iterative schemes (Jacobi, Gauss-Seidel, SOR). Convergence analysis.

6. Fundamentals of theory of matrice and vector spaces. Vector and matrix norms.

7. Problem of solving linear systems. Least square problem. Matrix factorizations: SVD, QR, LU, Cholesky, Jordan, Schur, etc. Basic matrix algorithms: Gram-Schmidt orthogonalizition, Householder triangularization, Gaussian elimination and pivoting.

8. Conditioning and stability theory.

9. Eigenvalue problem and singular value problem. Basic properties of eigenvalues/eigenvectors. Algorithms for eigen problem: power iteration, inverse iteration, Rayleigh quotient iteration, QR algorithm and shifts. Hessenberg decomposition.

10. Iterative methods for solving linear systems and eigenvalue problem: Arnoldi iteration, Lanczos iteration, conjugate gradient method.

III. References:

1. K. W. Morton and D. F. Mayers "Numerical Solution of Partial Differential Equations", Cambridge University Press, 1994.

2. R. D. Richtmyer and K. W. Morton, "Difference Methods for Initial Value Problems", Wiley-Interscience, 1967.

3. J. C. Strikwerda, "Finite Difference Schemes and Partial Differential Equations", Wadsworth & Brooks, 1989. SIAM 2004.

4. Lloyd N. Trefethen and David Bau, "Numerical Linear Algebra", SIAM 1997.

5. G. H. Golub and C. F. Van Loan, “Matrix Computations”, Johns Hopkins University Press, 1996.

6. J. W. Demmel, “Applied Numerical Linear Algebra”, SIAM 1997.

7. D. S. Watkins, “Fundamentals of Matrix Computations”, Wiley-Interscience, 1991, (2nd ed) 2002.

8. C William Gear, “Numerical Initial Value Problems in Ordinary Differential”, Prentice-Hall, 1971.

9. Peter Henrici, “Discrete Variable Methods in Ordinary Differential Equations”, Wiley, 1962.

10. C. Hall and T. Porsching, “Numerical Analysis of Partial Differential Equations”, Prentice Hall, 1990.

11. L. Shampine, “Numerical Solution of Ordinary Differential Equations”, Chapman & Hall, 1994.